Math 2130-101 **Test #2** **October 22^{nd}, 2012**

Name: ______________________________ **Be sure to show your work!**

If $F(x,y) = C$, then $\dfrac{dy}{dx} = -\dfrac{F_x}{F_y}$ If $F(x,y,z) = C$, then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z}$

1. (____/10 points) Let $f(x,y) = x^2 + 4y^2$.

(a) Write down a general equation for the level **curves** / **surfaces** of f (circle the correct term).

Briefly describe these level things.

(b) Write down the equation of the trace of $z = x^2 + 4y^2$ in the xz-plane.

(c) Make a rough sketch of $z = x^2 + 4y^2$.

2. (____/10 points) Consider x and y as independent variables and z as a dependent variable where $5x + xy^2 + ze^{3x+y} = 10$. Using the formulas for implicit differentiation, compute both $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

3. (____/10 points) Take it to the limit.

(a) Show $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x^2+2y^2+3z^2}{xy+yz+xz}$ does not exist.

(b) Show $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2+3y^2+5y^4}{x^2+y^2}$ exists (and find the limit). [*Hint:* Polar coordinates.]

4. (____/8 points) Consider the following statements:

A) The partial derivatives of $f(x,y)$ exist at $(x,y)=(a,b)$.

B) The partial derivatives of $f(x,y)$ are continuous at $(x,y)=(a,b)$.

C) $f(x,y)$ is continuous at $(x,y)=(a,b)$.

D) $f(x,y)$ is differentiable at $(x,y)=(a,b)$.

Write A, B, C, and D and draw arrows indicating implications between statements.

5. (_____/10 points) Let $f(x,y) = x^2y^2 + 3x^2 - 2y + 1$.

(a) Find ∇f (the gradient of f) and H_f (the Hessian matrix of f).

(b) Find the quadratic approximation of f centered at $(x,y) = (-1,2)$.

(c) Is $(-1,2)$ a critical point of f? Why or why not?

6. (_____/10 points) Suppose that $z = f(x,y)$, $x = u+v$, and $y = u-v$. Use the chain rule to show that

$$\frac{\partial z}{\partial u} \cdot \frac{\partial z}{\partial v} = \left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2$$

7. (____/10 points) Let $f(x,y) = x^3y - 2y + 1$.

(a) Find the directional derivative of f at the point $(-1,2)$ in the direction $\mathbf{v} = \langle 2, 1 \rangle$.

(b) What is the minimum value of $D_{\mathbf{u}}f(-1,2)$?

8. (____/12 points) Find and classify (determine if each is a relative min, relative max, or saddle point) the critical points of $f(x,y) = x^3 + \frac{3}{2}y^4 - 3xy^2$. [*Hint:* There are 3 points.]

9. (____/10 points) Let $xyz + x + 2y + 3z = 4$. Find the tangent plane to this surface at the point $(x, y, z) = (-2, 3, 0)$.

10. (____/10 points) Consider $f(x, y, z) = x^2y^3 + 6z$ subject to the constraint $xyz + x^2 + y^2 = 5$. Suppose we wished to find the maximum and minimum values of f subject to this constraint using the method of Lagrange multipliers. Find the equations which we would need to solve. *Note:* List **all** of the equations. Your answer should consist of scalar equations (no vector equations please). Finally, do **not** attempt to solve these equations!! (It will only end in tears.)

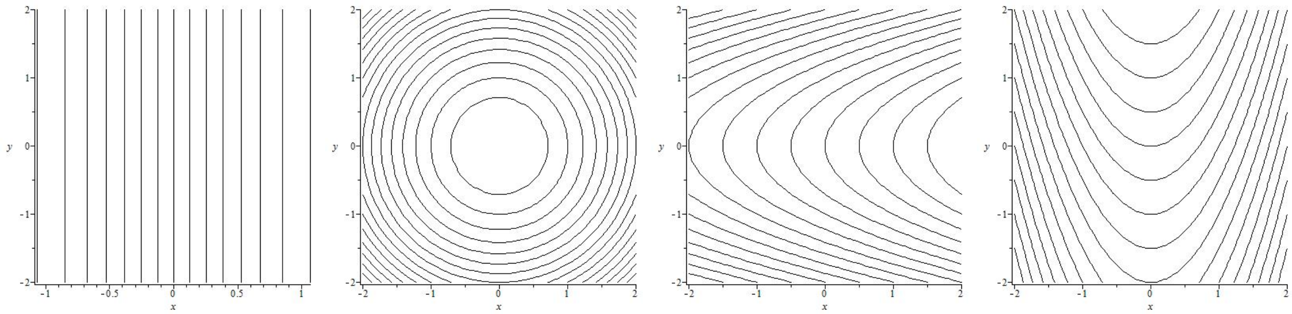
Math 2130-102 # Test #1 **October 22^{nd}, 2012**

Name: ______________________________ **Be sure to show your work!**

If $F(x,y)=C$, then $\dfrac{dx}{dy}=-\dfrac{F_x}{F_y}$ If $F(x,y,z)=C$, then $\dfrac{\partial z}{\partial x}=-\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y}=-\dfrac{F_y}{F_z}$

1. (____/10 points) Graphing

(a) Match each contour plots (plots of level curves) to its function.



A **B** **C** **D**

☐ $f(x,y)=y^2-x$ ☐ $f(x,y)=y-x^2$ ☐ $f(x,y)=x^2+y^2$ ☐ $f(x,y)=\sin(x)$

(b) Write down a general equation for the level surfaces of $g(x,y,z)=(x-1)^2+(y-2)^2+(z-3)^2$.

Briefly describe these level surfaces:

2. (____/10 points) Consider x and y as independent variables and z as a dependent variable where $xyz+\sin(3x+yz)+7x=12$. Using the formulas for implicit differentiation, compute both $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

3. (____/10 points) A Problem of Continuity.

(a) The function $f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ is not continuous at the origin. Show this is the case.

(b) The function $f(x,y) = \begin{cases} \dfrac{2x^2y}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ is continuous at the origin. Show this is the case.

[*Hint:* Polar coordinates.]

4. (____/8 points) Consider the following statements:

A) The partial derivatives of $f(x,y)$ exist at $(x,y) = (a,b)$.

B) The partial derivatives of $f(x,y)$ are continuous at $(x,y) = (a,b)$.

C) $f(x,y)$ is continuous at $(x,y) = (a,b)$.

D) $f(x,y)$ is differentiable at $(x,y) = (a,b)$.

Write A, B, C, and D and draw arrows indicating implications between statements.

5. (____/10 points) Let $f(x, y) = x^3y + 2y^2 - x + 1$.

(a) Find ∇f (the gradient of f) and H_f (the Hessian matrix of f).

(b) Find the quadratic approximation of f centered at $(x, y) = (1, -2)$.

(c) Is $(-1, 2)$ a critical point of f? Why or why not?
[Note: I meant to use the same point $(1, -2)$ as in the previous part, $(-1, 2)$ is essentially a typo.]

6. (____/8 points) State the chain rule for [~~$z = f(x, y, z)$~~] $w = f(x, y, z)$, $x = g(t)$, $y = h(t)$, and $z = k(t)$.

7. (____/10 points) Let $xy^2z^3 - 2x + y = 1$. Find the tangent plane to this surface at the point $(x, y, z) = (0, 1, -1)$.

8. (_____/10 points) Let $f(x, y) = x^2y^3 + 2$.

(a) Find the directional derivative of f at the point $(1, -1)$ in the direction $\mathbf{v} = \langle 3, 4 \rangle$.

(b) What is the maximum value of $D_{\mathbf{u}}f(1, -1)$?

9. (_____/12 points) Use the method of Lagrange multipliers to find the minimum and maximum **values** of $f(x, y, z) = xyz$ subject to the constraint $x^2 + 4y^2 + z^2 = 12$.

10. (____/12 points) A critical question.

(a) Let $f(1,2) = 0$, $f_x(1,2) = -1$, $f_y(1,2) = 0$, $f_{xx}(1,2) = 5$, $f_{xy}(1,2) = 3$, $f_{yx}(1,2) = -2$, and $f_{yy}(1,2) = 1$. Write down $\nabla f(1,2)$ and $H_f(1,2)$ (the gradient and Hessian of f at $(1,2)$). Is $(x,y) = (1,2)$ a critical point of f? If not, why not? If so, what kind of critical point is this (relative minimum, relative maximum, saddle point, or not enough information)?

(b) Let $f(1,2) = 7$, $f_x(1,2) = 0$, $f_y(1,2) = 0$, $f_{xx}(1,2) = 1$, $f_{xy}(1,2) = 2$, $f_{yx}(1,2) = 2$, and $f_{yy}(1,2) = 3$. Write down $\nabla f(1,2)$ and $H_f(1,2)$ (the gradient and Hessian of f at $(1,2)$). Is $(x,y) = (1,2)$ a critical point of f? If not, why not? If so, what kind of critical point is this (relative minimum, relative maximum, saddle point, or not enough information)?

(c) Let $f(1,2) = 3$, $f_x(1,2) = 0$, $f_y(1,2) = 0$, $f_{xx}(1,2) = -2$, $f_{xy}(1,2) = 3$, $f_{yx}(1,2) = 3$, and $f_{yy}(1,2) = -5$. Write down $\nabla f(1,2)$ and $H_f(1,2)$ (the gradient and Hessian of f at $(1,2)$). Is $(x,y) = (1,2)$ a critical point of f? If not, why not? If so, what kind of critical point is this (relative minimum, relative maximum, saddle point, or not enough information)?

(d) In part (a), there is something odd about the second partials. What is odd? What can be concluded from this?