Math 2130-101 | # Test #3 | **November 16^{th}, 2012**

Name: ________________________________ **Be sure to show your work!**

$$\begin{aligned} x &= \rho\cos(\theta)\sin(\varphi) \\ y &= \rho\sin(\theta)\sin(\varphi) \\ z &= \rho\cos(\varphi) \end{aligned}$$

$$J = \rho^2\sin(\varphi)$$

$$\cos^2(\theta) = \frac{1}{2}\left(1+\cos(2\theta)\right)$$

1. (14 points) Use a double Riemann sum to approximate $\displaystyle\iint_R y\ln(x^2+3)\,dA$ where $R = [-1,3]\times[-3,0]$. Using midpoint rule and a 2×3 grid of rectangles to partition R. (Don't worry about simplifying.)

2. (14 points) Consider $\displaystyle\iint_R x^2y\,dA$ where R is the region bounded by $y = 10-x^2$ and $y = x^2+2$. First, sketch the region of integration. Then set up (but do **not** evaluate) the integral in **both** orders of integration. *Hint:* The integral will have to be split into 2 pieces in one of the orders of integration.

3. (14 points) Evaluate $\iint\limits_R y\,dA$ where R is region inside $\dfrac{x^2}{9}+\dfrac{y^2}{16}=1$ with $y \geq 0$.

Hint: Use modified polar coordinates.

4. (15 points) Consider the integral: $I = \displaystyle\int_{-2}^{0}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{0} 10y^2\,dz\,dy\,dx.$

(a) Rewrite I in the following order of integration: $\iiint \quad dy\,dx\,dz.$
Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.
Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.
Do **not** evaluate the integral.

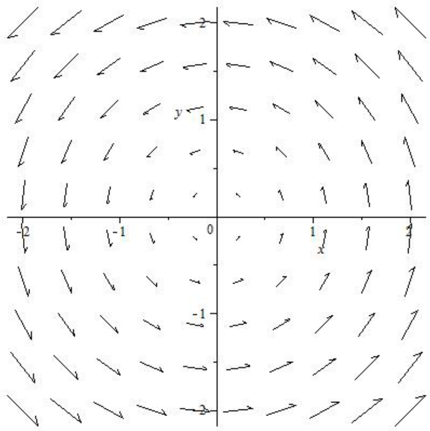
5. (14 points) Find the centroid of the region E where E is bounded below by $z = x^2 + y^2$ and above by $z = 4$. *Hint:* Use symmetry to cut down the number of integrals you need to evaluate. Also, note that the volume of E is 8π.

$$m = \iiint_E 1\,dV \qquad M_{yz} = \iiint_E x\,dV \qquad M_{xz} = \iiint_E y\,dV \qquad M_{xy} = \iiint_E z\,dV$$

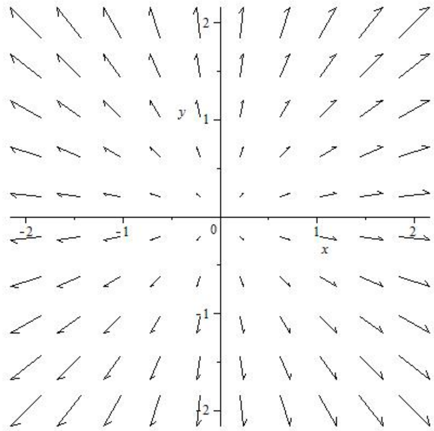
6. (14 points) Evaluate $\displaystyle\iiint_E \sqrt{x^2 + y^2 + z^2}\,dV$ where E is the region above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 9$.

7. (15 points) A few vector fields.

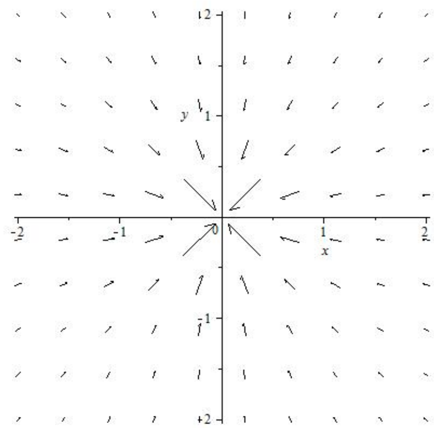
(a) The following are plots of several vector fields. Please note that all of the vectors have been scaled down so they do not overlap each other. Write A, B, and C next to the appropriate vector field's formula.



A



B



C

☐ $\mathbf{F}(x, y) = \left\langle \dfrac{-x}{x^2+y^2}, \dfrac{-y}{x^2+y^2} \right\rangle$ ☐ $\mathbf{F}(x, y) = \langle -y, x \rangle$ ☐ $\mathbf{F}(x, y) = \langle -y, -x \rangle$ ☐ $\mathbf{F}(x, y) = \langle x, y \rangle$

(b) Compute $\nabla \bullet \mathbf{F}$ and $\nabla \times \mathbf{F}$ where $\mathbf{F}(x, y, z) = \langle 2xy, x^2, \cos(z) \rangle$. Is $\mathbf{F}$ conservative? __________

(c) Compute $\nabla \bullet \mathbf{F}$ and $\nabla \times \mathbf{F}$ where $\mathbf{F}(x, y, z) = \langle e^{xyz}, x^2 + 1, x^2 z^3 \rangle$. Is $\mathbf{F}$ conservative? __________

Math 2130-102 **Test #3** **November 16^{th}, 2012**

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$$
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z &= \rho\cos(\varphi)
\end{aligned}
$$

$$J = \rho^2\sin(\varphi)$$

$$\cos^2(\theta) = \frac{1}{2}\left(1+\cos(2\theta)\right)$$

1. (14 points) Use a double Riemann sum to approximate $\iint\limits_R x^4 + 5e^y\,dA$ where $R = [-2,1]\times[-2,2]$. Using midpoint rule and a 3×2 grid of rectangles to partition R. (Don't worry about simplifying.)

2. (14 points) Sketch the region of integration and then evaluate $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+5}\,dx\,dy$.

   Hint: $\int \sqrt{x^3+5}\,dx$ isn't something we know how to integrate.

3. (14 points) Consider the integral $\displaystyle\iint_R e^{x+y}\cos(2x+5y)\,dA$ where R is bounded by $y=-x$, $y=-x+3$, $y=-\frac{2}{5}x-1$, and $y=-\frac{2}{5}x+2$. State a change of coordinates: $u=???$ and $v=???$ so that the resulting integral can be evaluated. Perform the change of coordinates and write down an iterated integral from which you could compute the answer. Do **not** evaluate your integral.

4. (15 points) Consider the integral: $I=\displaystyle\int_0^3\int_{-\sqrt{9-x^2}}^{0}\int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} 5(x^2+y^2+z^2)\,dz\,dy\,dx.$

(a) Rewrite I in the following order of integration: $\iiint \quad dy\,dx\,dz$.

Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.

Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.

Do **not** evaluate the integral.

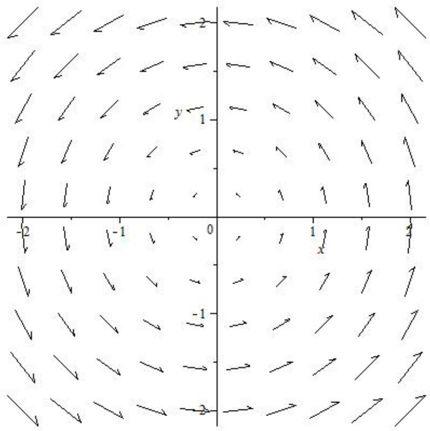
5. (14 points) Find the centroid of the region E where E is the region inside the unit sphere $x^2 + y^2 + z^2 = 1$ and in the first octant (i.e. $x, y, z \geq 0$). *Hint:* Use symmetry to cut down the number of integrals you need to evaluate. Recall that the volume of a sphere of radius R is $\frac{4}{3}\pi R^3$.

$$m = \iiint_E 1\, dV \qquad M_{yz} = \iiint_E x\, dV \qquad M_{xz} = \iiint_E y\, dV \qquad M_{xy} = \iiint_E z\, dV$$

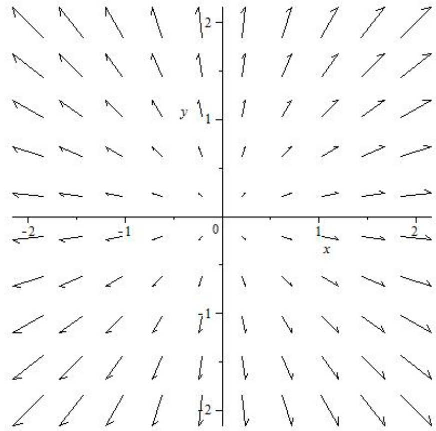
6. (14 points) Evaluate $\displaystyle\iiint\limits_E \sqrt{x^2 + y^2}\, dV$ where E is bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by $z = 3$.

7. (15 points) A few vector fields.

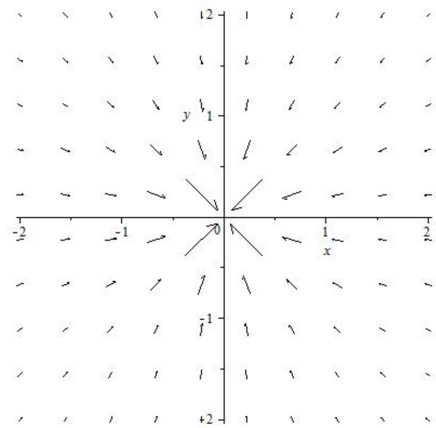
(a) The following are plots of several vector fields. Please note that all of the vectors have been scaled down so they do not overlap each other. Write A, B, and C next to the appropriate vector field's formula.



A



B



C

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(b) Compute $\nabla \bullet \mathbf{F}$ and $\nabla \times \mathbf{F}$ where $\mathbf{F}(x,y,z) = \langle 2xy, x^2, \cos(z) \rangle$. Is $\mathbf{F}$ conservative? ___________

(c) Compute $\nabla \bullet \mathbf{F}$ and $\nabla \times \mathbf{F}$ where $\mathbf{F}(x,y,z) = \langle e^{xyz}, x^2+1, x^2z^3 \rangle$. Is $\mathbf{F}$ conservative? ___________