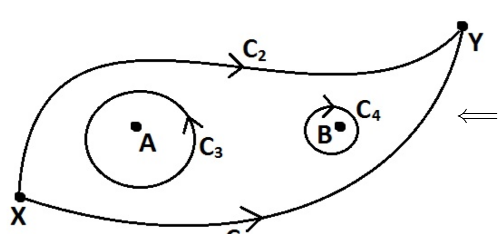
Name: ______________________________ **Be sure to show your work!**

1. (12 points) Let $\mathbf{F}(x, y, z) = \langle 2y + yz^2, 2x + xz^2 + 1, 2xyz + 3z^2 \rangle$.

(a) Compute $\int_C \mathbf{F} \bullet d\mathbf{r}$ where C is the line segment from $(-1, 0, 1)$ to $(2, 1, 1)$. Compute this line integral directly. [Do not use the fundamental theorem of line integrals for this part.]

(b) Show $\mathbf{F}$ is conservative and then use the fundamental theorem of line integrals to compute $\int_C \mathbf{F} \bullet d\mathbf{r}$.



$\Longleftarrow$ Both C_1 and C_2 begin at X and end at Y.

2. (6 points) Let $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y) \rangle$ be a vector field such that P and Q have continuous first partials and in addition, $P_y = Q_x$ everywhere except at the points A and B. Suppose that $\int_{C_2} \mathbf{F} \bullet d\mathbf{r} = 5$, $\int_{C_3} \mathbf{F} \bullet d\mathbf{r} = 10$, and $\int_{C_4} \mathbf{F} \bullet d\mathbf{r} = 3$.

Then $\int_{C_1} P(x, y)\, dx + Q(x, y)\, dy \quad = \quad$ ____________________.

3. (10 points) Applying the Divergence Theorem.

(a) Suppose that S_1 and S_2 are oriented smooth surfaces which share the same boundary C. In addition suppose that $S_1 - S_2$ is the outward oriented boundary of some simple solid region E. Finally, let $\mathbf{F}(x, y, z)$ be a vector field whose component functions have continuous partials (i.e. a "nice" vector field).

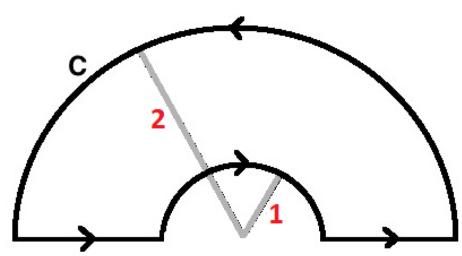
Use the divergence theorem to write down an equation relating $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$ and $\iint_{S_2} \mathbf{F} \bullet d\mathbf{S}$.

This demonstrates that if the divergence of $\mathbf{F}$ is 0, then we will have ________________________________.

(b) Suppose S_1 is the upper-half of the sphere $x^2 + y^2 + z^2 = 1$ $(z \geq 0)$ oriented upward. Let S_2 be the unit disk in the xy-plane $(x^2 + y^2 \leq 1)$ oriented upward. Suppose we know that $\iint_{S_2} \mathbf{F} \bullet d\mathbf{S} = 5$. In addition, we know that $\nabla \bullet \mathbf{F} = 3$.

Find $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$.

4. (11 points) Let C be the boundary of the upper-half of the annulus centered at the origin with inner radius 1 and outer radius 2 oriented counter-clockwise.



Find $\displaystyle\int_C \left(e^{-x^3+77x} - y^3\right)\, dx + \left(\frac{1}{\sqrt[3]{y^5+9}} + x^3\right)\, dy$

5. (12 points) Find the centroid of C where C is parameterized by $\mathbf{r}(t) = \langle 3\cos(t), 4t, 3\sin(t) \rangle$, $0 \le t \le 2\pi$.
[*Note:* You must work out these line integrals. I don't want answers via symmetry.]

$$m = \int_C ds \qquad M_{yz} = \int_C x\,ds \qquad M_{xz} = \int_C y\,ds \qquad M_{xy} = \int_C z\,ds$$

6. (12 points) Find the centroid of the of the part of the unit sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant (i.e. $x, y, z \ge 0$). Please use geometry and symmetry to cut down the number of **surface** integrals you need to compute. You are dealing with **one-eighth** of the unit sphere.

$$m = \iint_C dS \qquad M_{yz} = \iint_C x\,dS \qquad M_{xz} = \iint_C y\,dS \qquad M_{xy} = \iint_C z\,dS$$

7. (13 points) Let S_1 be the surface parameterized by $\mathbf{r}(u,v) = \langle u\cos(v), u\sin(v), 4-u^2\rangle$ where $1 \le u \le 2$ and $0 \le v \le 2\pi$.

(a) Find both orientations for S_1.

(b) Set up but **do not evaluate** the surface integral $\iint_{S_1} ((z-2x)e^y)\, dS$. [Don't worry about simplifying.]

(c) Set up but **do not evaluate** the flux integral $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$ where S_1 is oriented downward and $\mathbf{F}(x,y,z) = \langle z, x+y, x^3\rangle$.
[Don't worry about computing the dot product or any significant simplifying.]

8. (11 points) Let E be solid bounded below by $z = x^2 + y^2$ and above by $z = 4$ and let S_1 be the surface of E oriented outward and let $\mathbf{F}(x,y,z) = \langle xy^2, yx^2, 2\rangle$. Compute $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$. *Hint:* S_1 is closed surface bounding the solid region E.

9. (13 points) Let C be the circle $x^2 + y^2 = 9$ where $z = 2$ (a circle of radius 3 parallel to the xy-plane and centered at $(0, 0, 2)$). Orient C counter-clockwise when viewed from above. Verify Stokes' Theorem for S_1 (the disk whose boundary is C) and the vector field $\mathbf{F}(x, y, z) = \langle y, yz, z \rangle$.

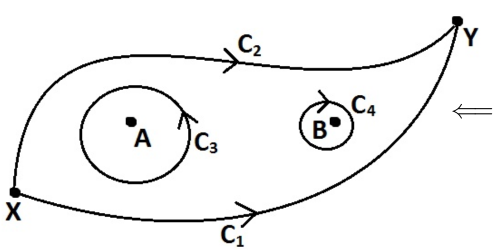
Math 2130-101 **Final Exam** December 12^{th}, 2013

Name: ____________________________________ **Be sure to show your work!**

1. (12 points) Let $\mathbf{F}(x, y, z) = \langle y + yz^2 + 1, x + xz^2, 2xyz + e^z \rangle$.

(a) Compute $\int_C \mathbf{F} \bullet d\mathbf{r}$ where C is the line segment from $(1, 0, 2)$ to $(4, 1, 2)$. Compute this line integral directly. [Do not use the fundamental theorem of line integrals for this part.]

(b) Show $\mathbf{F}$ is conservative and then use the fundamental theorem of line integrals to compute $\int_C \mathbf{F} \bullet d\mathbf{r}$.



$\Longleftarrow$ Both C_1 and C_2 begin at X and end at Y.

2. (6 points) Let $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y) \rangle$ be a vector field such that P and Q have continuous first partials and in addition, $P_y = Q_x$ everywhere except at the points A and B. Suppose that $\int_{C_2} \mathbf{F} \bullet d\mathbf{r} = 6$, $\int_{C_3} \mathbf{F} \bullet d\mathbf{r} = 4$, and $\int_{C_4} \mathbf{F} \bullet d\mathbf{r} = 5$.

Then $\int_{C_1} P(x, y)\, dx + Q(x, y)\, dy \quad = \quad$ ____________________.

3. (10 points) Applying the Divergence Theorem.

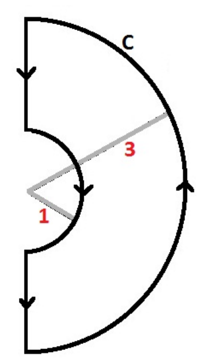
(a) Suppose that S_1 and S_2 are oriented smooth surfaces which share the same boundary C. In addition suppose that $S_1 - S_2$ is the outward oriented boundary of some simple solid region E. Finally, let $\mathbf{F}(x, y, z)$ be a vector field whose component functions have continuous partials (i.e. a "nice" vector field).

Use the divergence theorem to write down an equation relating $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$ and $\iint_{S_2} \mathbf{F} \bullet d\mathbf{S}$.

This demonstrates that if the divergence of $\mathbf{F}$ is 0, then we will have ______________________________.

(b) Suppose S_1 is the upper-half of the sphere $x^2 + y^2 + z^2 = 1$ ($z \geq 0$) oriented upward. Let S_2 be the unit disk in the xy-plane ($x^2 + y^2 \leq 1$) oriented upward. Suppose we know that $\iint_{S_2} \mathbf{F} \bullet d\mathbf{S} = 10$. In addition, we know that $\nabla \bullet \mathbf{F} = 6$.

Find $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$.



4. (11 points) Let C be the boundary of the right-half of the annulus centered at the origin with inner radius 1 and outer radius 3. Also, C is oriented counter-clockwise.

Find $\int_C \left(e^{-x^3+77x} - xy\right) dx + \left(\frac{1}{\sqrt[3]{y^5+9}}\right) dy$

5. (12 points) Find the centroid of C where C is parameterized by $\mathbf{r}(t) = \langle 4t, 3\cos(t), 3\sin(t) \rangle$, $0 \le t \le 2\pi$.
[*Note:* You must work out these line integrals. I don't want answers via symmetry.]

$$m = \int_C ds \qquad M_{yz} = \int_C x\, ds \qquad M_{xz} = \int_C y\, ds \qquad M_{xy} = \int_C z\, ds$$

6. (12 points) Find the centroid of the of the lower-half of the unit sphere $x^2 + y^2 + z^2 = 1$ (and $z \le 0$). Please use geometry and symmetry to cut down the number of **surface** integrals you need to compute.

$$m = \iint_C dS \qquad M_{yz} = \iint_C x\, dS \qquad M_{xz} = \iint_C y\, dS \qquad M_{xy} = \iint_C z\, dS$$

7. (13 points) Let S_1 be the surface parameterized by $\mathbf{r}(u,v) = \langle u\cos(v), u\sin(v), 3-u\rangle$ where $1 \le u \le 3$ and $0 \le v \le 2\pi$.

(a) Find both orientations for S_1.

(b) Set up but **do not evaluate** the surface integral $\iint_{S_1} (x^2+y)\sin(z)\, dS$. [Don't worry about simplifying.]

(c) Set up but **do not evaluate** the flux integral $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$ where S_1 is oriented downward and $\mathbf{F}(x,y,z) = \langle y^2, z, xy\rangle$. [Don't worry about computing the dot product or any significant simplifying.]

8. (11 points) Let E be solid bounded below by $z = x^2 + y^2$ and above by $z = 4$ and let S_1 be the surface of E oriented outward and let $\mathbf{F}(x,y,z) = \langle xy^2, yx^2, 2\rangle$. Compute $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$. *Hint:* S_1 is closed surface bounding the solid region E.

9. (13 points) Let C be the circle $x^2 + y^2 = 9$ where $z = 2$ (a circle of radius 3 parallel to the xy-plane and centered at $(0, 0, 2)$). Orient C counter-clockwise when viewed from above. Verify Stokes' Theorem for S_1 (the disk whose boundary is C) and the vector field $\mathbf{F}(x, y, z) = \langle y, yz, z\rangle$.