Math 2130-101 **Test #1** **September 20^{th}, 2013**

Name: Answer Key **Be sure to show your work!**

1. (18 points) Vector Basics

(a) Find the volume of the parallelepiped spanned by $\mathbf{u} = \langle 1, 2, -1\rangle$, $\mathbf{v} = \langle 1, 0, 1\rangle$, and $\mathbf{w} = \langle -1, 2, 2\rangle$.

$$\begin{array}{rrrrcl} & \mathbf{i} & \mathbf{j} & \mathbf{k} & & \\ & 1 & 2 & -1 & = & \mathbf{u} \\ \times & 1 & 0 & 1 & = & \mathbf{v} \\ \hline & 2 & -2 & -2 & = & \mathbf{u}\times\mathbf{v} \end{array} \implies (\mathbf{u}\times\mathbf{v})\bullet\mathbf{w} = \langle 2,-2,-2\rangle \bullet \langle -1,2,2\rangle = -10 \implies \text{Volume} = \boxed{10}$$

OR

$$\begin{array}{rrrrcl} & \mathbf{i} & \mathbf{j} & \mathbf{k} & & \\ & 1 & 0 & 1 & = & \mathbf{v} \\ \times & -1 & 2 & 2 & = & \mathbf{w} \\ \hline & -2 & -3 & 2 & = & \mathbf{v}\times\mathbf{w} \end{array} \implies \mathbf{u}\bullet(\mathbf{v}\times\mathbf{w}) = \langle 1,2,-1\rangle \bullet \langle -2,-3,2\rangle = -10 \implies \text{Volume} = \boxed{10}$$

OR

$$\begin{vmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{w} \end{vmatrix} = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ -1 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 0 & 1 \\ 2 & 2 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} = 1(-2) - 2(3) - 1(2) = -10 \implies \text{Volume} = \boxed{10}$$

(b) Find the angle between $\mathbf{u} = \langle 3, 2, 1\rangle$ and $\mathbf{w} = \langle 0, 1, 1\rangle$ (don't worry about evaluating inverse trig. functions).

$$\begin{array}{rclcl} \mathbf{u}\bullet\mathbf{w} & = & 3(0)+2(1)+1(1) & = & 3 \\ |\mathbf{u}| & = & \sqrt{3^2+2^2+1^2} & = & \sqrt{14} \\ |\mathbf{w}| & = & \sqrt{0^2+1^2+1^2} & = & \sqrt{2} \end{array} \implies \theta = \arccos\left(\frac{3}{\sqrt{14}\sqrt{2}}\right) = \boxed{\arccos\left(\frac{3}{2\sqrt{7}}\right)}$$

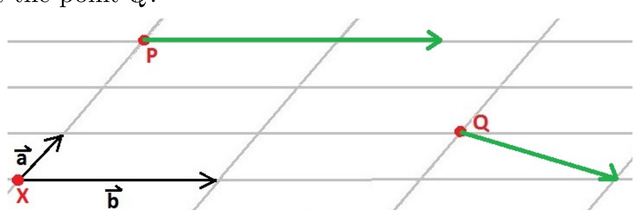
Is this angle... **right**, $\boxed{\textbf{acute}}$, or **obtuse** ? (Circle your answer.) [Because $\mathbf{u}\bullet\mathbf{w} > 0$.]

(c) Fill in the blanks...

$|\mathbf{a}\times\mathbf{b}|$ computes the __area__ of the __parallelogram__ spanned by **a** and **b**.

$|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$ computes the __volume__ of the __parallelepiped__ spanned by **a**, **b**, and **c**.

(d) The vectors **a** and **b** are shown below. They are based a the point X. Sketch the vector $1.5\mathbf{b}$ based at the point P and sketch the vector $\mathbf{b}-\mathbf{a}$ based at the point Q.



2. (9 points)

Let ℓ_1 be parametrized by $\mathbf{r}_1(t) = \langle 1, 0, -1\rangle + \langle 1, 2, -1\rangle t$ and ℓ_2 by $\mathbf{r}_2(t) = \langle 1, -4, 1\rangle + \langle -2, -4, 2\rangle t$. Determine if ℓ_1 and ℓ_2 are... (circle the correct answer)

the same, $\boxed{\text{parallel}}$ (but not the same), intersecting, or skew.

First, notice that $-2\mathbf{r}_1'(t) = -2\langle 1, 2, -1\rangle = \langle -2, -4, 2\rangle = \mathbf{r}_2'(t)$, so these lines have parallel direction vectors. Thus our lines are either parallel or the same.

Let's see if they intersect: $\mathbf{r}_1(t) = \mathbf{r}_2(s)$ so that $1+t = 1-2s$, $2t = -4-4s$, and $-1-t = 1+2s$. Using the second equation, $t = -2-2s$. Plugging this into the first equation gives us $1+(-2-2s) = 1-2s$ so that $-1 = 1$. Since $-1 \neq 1$, this is an inconsistent system and there is no solution. These lines do not intersect, so they are distinct parallel lines.

3. (14 points) Plane old geometry.

(a) Find the (scalar) equation of the plane through the points $A = (2, 2, 1)$, $B = (3, 2, 1)$, and $C = (4, 3, -1)$.

$$\begin{array}{rrrrcccl} & \mathbf{i} & \mathbf{j} & \mathbf{k} & & & & \\ & 1 & 0 & 0 & = & B-A & = & \vec{AB} \\ \times & 2 & 1 & -2 & = & C-A & = & \vec{AC} \\ \hline & 0 & 2 & 1 & = & & \mathbf{n} & \end{array} \implies 0(x-2)+2(y-2)+1(z-1) = 0 \implies \boxed{2y+z=5}$$

Here we found 2 vectors parallel to the plane (i.e. $\vec{AB}$ and $\vec{AC}$), found their cross product to get a normal, and then fit the plane through one of the points (i.e. A).

(b) Find the area of the triangle ΔABC (the triangle with vertices A, B, and C).

We already found vectors $\vec{AB}$ and $\vec{AC}$ which span a parallelogram whose area is twice that of ΔABC. In fact, this parallelogram's area is $|\vec{AB} \times \vec{AC}| = |\mathbf{n}| = |\langle 0,2,1\rangle| = \sqrt{5}$. Therefore, the area of ΔABC is $\boxed{\dfrac{\sqrt{5}}{2}}$.

4. (7 points) Find a formula for the curvature of $y = \sin(x)$.

Use the special formula for curvature of a graph in $\mathbb{R}^2$: $\quad \kappa(x) = \dfrac{|f''(x)|}{(1+(f'(x))^2)^{3/2}} = \dfrac{|\sin(x)|}{(1+\cos^2(x))^{3/2}}$

5. (18 points) Let C be the **right half** of the circle $x^2 + y^2 = 4$.

(a) Parameterize C and find a parameterization for the line tangent to C at $(\sqrt{2}, \sqrt{2})$.

We parameterize the circle using $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t)\rangle$. To get the right-half of the circle we restrict t to $-\pi/2 \le t \le \pi/2$. Of course, there are many other possible (correct) answers.

Next, $\mathbf{r}'(t) = \langle -2\sin(t), 2\cos(t)\rangle$. Notice that $\mathbf{r}(t) = \langle \sqrt{2}, \sqrt{2}\rangle$ implies that $2\cos(t) = \sqrt{2}$ and $2\sin(t) = \sqrt{2}$. Therefore, at this point $\mathbf{r}'(t) = \langle -\sqrt{2}, \sqrt{2}\rangle$. Alternatively, we could note that $\cos(t) = \dfrac{1}{\sqrt{2}}$ and $\sin(t) = \dfrac{1}{\sqrt{2}}$ means $t = \pi/4$. Therefore, $\mathbf{r}'(\pi/4) = \langle -2\sin(\pi/4), 2\cos(\pi/4)\rangle = \langle -2\frac{1}{\sqrt{2}}, 2\frac{1}{\sqrt{2}}\rangle = \langle -\sqrt{2}, \sqrt{2}\rangle$. Therefore, the tangent line is parameterized by $\mathbf{L}(t) = \langle \sqrt{2}, \sqrt{2}\rangle + \langle -\sqrt{2}, \sqrt{2}\rangle t$.

(b) Find the centroid of C.

$m = \int_C 1\,ds =$ arc length of half a circle of radius $2 = 2\pi$ and $\bar{y} = 0$ (by symmetry). We need to compute $M_y = \int_C x\,ds$. First, we need $ds = |\mathbf{r}'(t)|\,dt = \sqrt{4\sin^2(t) + 4\cos^2(t)}\,dt = 2\,dt$. So $M_y = \int_{-\pi/2}^{\pi/2} 2\cos(t)\cdot 2\,dt = 4\int_{-\pi/2}^{\pi/2}\cos(t)\,dt = 8$. Therefore, $\bar{x} = \dfrac{M_y}{m} = \dfrac{8}{2\pi} = \dfrac{4}{\pi}$. $\boxed{(\bar{x}, \bar{y}) = \left(\dfrac{4}{\pi}, 0\right)}$

6. (10 points) Find the curvature of the curve parameterized by $\mathbf{r}(t) = \langle e^t, t^2+1, \sin(t)\rangle$.
[Don't try to simplify your answer.]

We should use the formula for curvature with the cross product.

	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$		
	e^t	$2t$	$\cos(t)$	$=$	$\mathbf{r}'(t)$
$\times$	e^t	2	$-\sin(t)$	$=$	$\mathbf{r}''(t)$
	$-2t\sin(t) - 2\cos(t)$	$e^t\sin(t) + e^t\cos(t)$	$2e^t - 2te^t$	$=$	$\mathbf{r}'(t)\times\mathbf{r}''(t)$

$$\kappa(t) = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{\sqrt{(-2t\sin(t) - 2\cos(t))^2 + (e^t\sin(t) + e^t\cos(t))^2 + (2e^t - 2te^t)^2}}{(e^{2t} + 4t^2 + \cos^2(t))^{3/2}}$$

7. (12 points) Find the TNB-frame for $\mathbf{r}(t) = \langle \cos(t), t, \sin(t)\rangle$.

$$\mathbf{r}'(t) = \langle -\sin(t), 1, \cos(t)\rangle \implies |\mathbf{r}'(t)| = \sqrt{\sin^2(t) + 1^2 + \cos^2(t)} = \sqrt{2} \implies \mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{\sqrt{2}}\langle -\sin(t), 1, \cos(t)\rangle$$

$$\mathbf{T}'(t) = \frac{1}{\sqrt{2}}\langle -\cos(t), 0, -\sin(t)\rangle \implies |\mathbf{T}'(t)| = \frac{1}{\sqrt{2}}\sqrt{\cos^2(t) + 0^2 + \sin^2(t)} = \frac{1}{\sqrt{2}}$$

$$\implies \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\langle -\cos(t), 0, -\sin(t)\rangle = \langle -\cos(t), 0, -\sin(t)\rangle$$

	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$		
	$-\frac{1}{\sqrt{2}}\sin(t)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}\cos(t)$	$=$	$\mathbf{T}(t)$
$\times$	$-\cos(t)$	0	$-\sin(t)$	$=$	$\mathbf{N}(t)$
	$-\frac{1}{\sqrt{2}}\sin(t)$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}\cos(t)$	$=$	$\mathbf{T}(t)\times\mathbf{N}(t)$

$$\implies \mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t) = \frac{1}{\sqrt{2}}\langle -\sin(t), -1, \cos(t)\rangle$$

8. (12 points) No numbers here. Choose **ONE** of the following:

I. Suppose that $\mathbf{a}$ and $\mathbf{b}$ are both orthogonal to $\mathbf{c}$. Show that $2\mathbf{a} - \mathbf{b}$ is orthogonal to $\mathbf{c}$.

Since $\mathbf{a}$ and $\mathbf{b}$ are orthogonal to $\mathbf{c}$, $\mathbf{a} \bullet \mathbf{c} = 0$ and $\mathbf{b} \bullet \mathbf{c} = 0$. Therefore, $(2\mathbf{a} - \mathbf{b}) \bullet \mathbf{c} = (2\mathbf{a}) \bullet \mathbf{c} - \mathbf{b} \bullet \mathbf{c} = 2(\mathbf{a} \bullet \mathbf{c}) - (\mathbf{b} \bullet \mathbf{c}) = 2(0) - 0 = 0$. So $2\mathbf{a} - \mathbf{b}$ is orthogonal to $\mathbf{c}$.

II. Prove Lagrange's identity: $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a} \bullet \mathbf{b})^2$.

Doing this using components is a nightmare. Instead let's use the geometric formulas we found in class. Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 \sin^2(\theta) = |\mathbf{a}|^2|\mathbf{b}|^2(1 - \cos^2(\theta)) = |\mathbf{a}|^2|\mathbf{b}|^2 - |\mathbf{a}|^2|\mathbf{b}|^2 \cos^2(\theta) = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a} \bullet \mathbf{b})^2$.

Math 2130-102 **Test #1** **September 20^{th}, 2013**

Name: Answer Key **Be sure to show your work!**

1. (18 points) Vector Basics

(a) Find the volume of the parallelepiped spanned by $\mathbf{u} = \langle 2, 0, -1 \rangle$, $\mathbf{v} = \langle 1, 1, 2 \rangle$, and $\mathbf{w} = \langle 1, 2, 1 \rangle$.

Just like section 101's #1(a). **Answer:** Volume = $\boxed{7}$

(b) Find the angle between $\mathbf{u} = \langle -1, 2, 1 \rangle$ and $\mathbf{w} = \langle 3, 0, 1 \rangle$ (don't worry about evaluating inverse trig. functions).

Just like section 101's #1(b). **Answer:** $\theta = \arccos\left(\dfrac{-2}{\sqrt{6}\sqrt{10}}\right) = \boxed{\arccos\left(\dfrac{-1}{\sqrt{15}}\right)}$

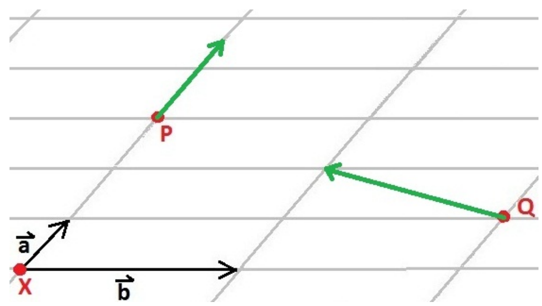
Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.) [Because $\mathbf{u} \bullet \mathbf{w} < 0$.]

(c) Fill in the blanks...

$|\mathbf{a} \times \mathbf{b}|$ computes the area of the parallelogram spanned by **a** and **b**.

$|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$ computes the volume of the parallelepiped spanned by **a**, **b**, and **c**.

(d) The vectors **a** and **b** are shown below. They are based a the point X. Sketch the vector $1.5\mathbf{a}$ based at the point P and sketch the vector $\mathbf{a} - \mathbf{b}$ based at the point Q.



2. (9 points) Let ℓ_1 be parametrized by $\mathbf{r}_1(t) = \langle 1, 0, -1 \rangle + \langle 1, 2, -1 \rangle t$ and ℓ_2 by $\mathbf{r}_2(t) = \langle 1, 4, 1 \rangle + \langle 1, -2, 2 \rangle t$. Determine if ℓ_1 and ℓ_2 are... (circle the correct answer)

the same, parallel (but not the same), intersecting, or skew.

First, notice that $\mathbf{r}_1'(t) = \langle 1, 2, -1 \rangle$ is not a scalar multiple of $\mathbf{r}_2'(t) = \langle 1, -2, 2 \rangle$, so these lines do not have parallel direction vectors. Thus our lines are either intersecting or skew.

Let's see if they intersect: $\mathbf{r}_1(t) = \mathbf{r}_2(s)$ so that $1 + t = 1 + s$, $2t = 4 - 2s$, and $-1 - t = 1 + 2s$. The first equation gives us $t = s$. Plugging this into the second equation, we get $2t = 4 - 2t$ so that $4t = 4$ so $t = 1$. Thus $s = t = 1$. Plugging this into the third equation yields $-1 - 1 = 1 + 2$ so $-2 = 3$. Since $-2 \neq 3$, this is an inconsistent system and there is no solution. These lines do not intersect, so they are distinct skew lines.

3. (14 points) Plane old geometry.

(a) Find the (scalar) equation of the plane through the points $A = (1, 2, 2)$, $B = (2, 2, 4)$, and $C = (-1, 3, 3)$.

$$\begin{array}{rrrrcccc} & \mathbf{i} & \mathbf{j} & \mathbf{k} & & & & \\ & 1 & 0 & 2 & = & B - A & = & \vec{AB} \\ \times & -2 & 1 & 1 & = & C - A & = & \vec{AC} \\ \hline & -2 & -5 & 1 & = & \mathbf{n} & & \end{array} \implies -2(x-1) - 5(y-2) + 1(z-2) = 0 \implies \boxed{-2x - 5y + z = -10}$$

Here we found 2 vectors parallel to the plane (i.e. $\vec{AB}$ and $\vec{AC}$), found their cross product to get a normal, and then fit the plane through one of the points (i.e. A).

(b) Find the area of the triangle ΔABC (the triangle with vertices A, B, and C).

We already found vectors $\vec{AB}$ and $\vec{AC}$ which span a parallelogram whose area is twice that of ΔABC. In fact, this parallelogram's area is $|\vec{AB} \times \vec{AC}| = |\mathbf{n}| = |\langle -2, -5, 1 \rangle| = \sqrt{30}$. Therefore, the area of ΔABC is $\boxed{\dfrac{\sqrt{30}}{2}}$.

4. (7 points) Find a formula for the curvature of $y = x^3$.

Use the special formula for curvature of a graph in $\mathbb{R}^2$: $\kappa(x) = \dfrac{|f''(x)|}{(1+(f'(x))^2)^{3/2}} = \dfrac{|6x|}{(1+9x^4)^{3/2}}$

5. (18 points) Let C be the **bottom half** of the circle $x^2 + y^2 = 4$.

(a) Parameterize C and find a parameterization for the line tangent to C at $(\sqrt{2}, -\sqrt{2})$.

We parameterize the circle using $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t)\rangle$. To get the bottom of the circle we restrict t to $\pi \le t \le 2\pi$. Of course, there are many other possible (correct) answers.

Next, $\mathbf{r}'(t) = \langle -2\sin(t), 2\cos(t)\rangle$. Notice that $\mathbf{r}(t) = \langle\sqrt{2}, -\sqrt{2}\rangle$ implies that $2\cos(t) = \sqrt{2}$ and $2\sin(t) = -\sqrt{2}$. Therefore, at this point $\mathbf{r}'(t) = \langle -(-\sqrt{2}), \sqrt{2}\rangle = \langle\sqrt{2}, \sqrt{2}\rangle$. Alternatively, we could note that $\cos(t) = \dfrac{1}{\sqrt{2}}$ and $\sin(t) = -\dfrac{1}{\sqrt{2}}$ means $t = 7\pi/4$ (equivalent to $-\pi/4$). Therefore, $\mathbf{r}'(7\pi/4) = \langle -2\sin(7\pi/4), 2\cos(7\pi/4)\rangle = \langle -2\frac{-1}{\sqrt{2}}, 2\frac{1}{\sqrt{2}}\rangle = \langle\sqrt{2}, \sqrt{2}\rangle$. Therefore, the tangent line is parameterized by $\mathbf{L}(t) = \langle\sqrt{2}, -\sqrt{2}\rangle + \langle\sqrt{2}, \sqrt{2}\rangle t$.

(b) Find the centroid of C.

$m = \int_C 1\,ds =$ arc length of half a circle of radius $2 = 2\pi$ and $\bar{x} = 0$ (by symmetry). We need to compute $M_x = \int_C y\,ds$. First, we need $ds = |\mathbf{r}'(t)|\,dt = \sqrt{4\sin^2(t) + 4\cos^2(t)}\,dt = 2\,dt$. So $M_x = \int_\pi^{2\pi} 2\sin(t)\cdot 2\,dt = 4\int_{-\pi}^{2\pi}\sin(t)\,dt = -8$. Therefore, $\bar{y} = \dfrac{M_y}{m} = \dfrac{-8}{2\pi} = -\dfrac{4}{\pi}$. $\boxed{(\bar{x}, \bar{y}) = \left(0, -\dfrac{4}{\pi}\right)}$

6. (10 points) Find the curvature of the curve parameterized by $\mathbf{r}(t) = \langle\cos(t), e^t + 1, t^2\rangle$.
[Don't try to simplify your answer.]

We should use the formula for curvature with the cross product.

	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$		
	$-\sin(t)$	e^t	$2t$	$=$	$\mathbf{r}'(t)$
$\times$	$-\cos(t)$	e^t	2	$=$	$\mathbf{r}''(t)$
	$2e^t - 2te^t$	$2\sin(t) - 2t\cos(t)$	$-e^t\sin(t) + e^t\cos(t)$	$=$	$\mathbf{r}'(t)\times\mathbf{r}''(t)$

$$\kappa(t) = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{\sqrt{(2e^t - 2te^t)^2 + (2\sin(t) - 2t\cos(t))^2 + (-e^t\sin(t) + e^t\cos(t))^2}}{(\sin^2(t) + e^{2t} + 4t^2)^{3/2}}$$

7. (12 points) Find the TNB-frame for $\mathbf{r}(t) = \langle t, \sin(t), \cos(t)\rangle$.

$$\mathbf{r}'(t) = \langle 1, \cos(t), -\sin(t)\rangle \implies |\mathbf{r}'(t)| = \sqrt{1^2 + \cos^2(t) + \sin^2(t)} = \sqrt{2} \implies \mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{\sqrt{2}}\langle 1, \cos(t), -\sin(t)\rangle$$

$$\mathbf{T}'(t) = \frac{1}{\sqrt{2}}\langle 0, -\sin(t), -\cos(t)\rangle \implies |\mathbf{T}'(t)| = \frac{1}{\sqrt{2}}\sqrt{0^2 + \sin^2(t) + \cos^2(t)} = \frac{1}{\sqrt{2}}$$

$$\implies \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\langle 0, -\sin(t), -\cos(t)\rangle = \langle 0, -\sin(t), -\cos(t)\rangle$$

	$\mathbf{i}$	$\mathbf{j}$	$\mathbf{k}$		
	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}\cos(t)$	$-\frac{1}{\sqrt{2}}\sin(t)$	$=$	$\mathbf{T}(t)$
$\times$	0	$-\sin(t)$	$-\cos(t)$	$=$	$\mathbf{N}(t)$
	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}\cos(t)$	$-\frac{1}{\sqrt{2}}\sin(t)$	$=$	$\mathbf{T}(t)\times\mathbf{N}(t)$

$$\implies \mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t) = \frac{1}{\sqrt{2}}\langle -1, \cos(t), -\sin(t)\rangle$$

8. (12 points) No numbers here. Choose **ONE** of the following:

I. Suppose $|\mathbf{r}(t)| = c$ ($\mathbf{r}(t)$ has constant length) for all t. Show that $\mathbf{r}(t)$ is orthogonal to $\mathbf{r}'(t)$.

$$0 = \frac{d}{dt}\left[c^2\right] = \frac{d}{dt}\left[|\mathbf{r}(t)|^2\right] = \frac{d}{dt}\left[\mathbf{r}(t)\bullet\mathbf{r}(t)\right] = \mathbf{r}'(t)\bullet\mathbf{r}(t) + \mathbf{r}(t)\bullet\mathbf{r}'(t) = 2\mathbf{r}(t)\bullet\mathbf{r}'(t) \implies \mathbf{r}(t)\bullet\mathbf{r}'(t) = 0$$

II. Compute $(\mathbf{a}+\mathbf{b})\bullet(\mathbf{a}\times\mathbf{b})$. Then give a geometric interpretation of your computation.

$(\mathbf{a}+\mathbf{b})\bullet(\mathbf{a}\times\mathbf{b}) = \mathbf{a}\bullet(\mathbf{a}\times\mathbf{b}) + \mathbf{b}\bullet(\mathbf{a}\times\mathbf{b}) = 0 + 0 = 0$ since $\mathbf{a}$ and $\mathbf{b}$ are orthogonal to $\mathbf{a}\times\mathbf{b}$. This means that $\mathbf{a}+\mathbf{b}$ is perpendicular to $\mathbf{a}\times\mathbf{b}$. Another way to approach this problem: $(\mathbf{a}+\mathbf{b})\bullet(\mathbf{a}\times\mathbf{b}) = 0$ since we have a triple scalar product of coplanar vectors. We have that $\mathbf{a}+\mathbf{b}$, $\mathbf{a}$, and $\mathbf{b}$ are coplanar since $\mathbf{a}+\mathbf{b}$ is a linear combination of $\mathbf{a}$ and $\mathbf{b}$.