Math 2130-101 **Test #1** **September 20$^{\text{th}}$, 2013**

Name: ______________________________ **Be sure to show your work!**

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \qquad \mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t) \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$m = \int_C \rho\, ds \qquad (\bar{x}, \bar{y}, \bar{z}) = \frac{1}{m}\left(\int_C x\rho\, ds, \int_C y\rho\, ds, \int_C z\rho\, ds\right) \qquad \kappa = \frac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{\frac{3}{2}}}$$

1. (18 points) Vector Basics

(a) Find the volume of the parallelepiped spanned by $\mathbf{u} = \langle 1, 2, -1 \rangle$, $\mathbf{v} = \langle 1, 0, 1 \rangle$, and $\mathbf{w} = \langle -1, 2, 2 \rangle$.

(b) Find the angle between $\mathbf{u} = \langle 3, 2, 1 \rangle$ and $\mathbf{w} = \langle 0, 1, 1 \rangle$ (don't worry about evaluating inverse trig. functions).

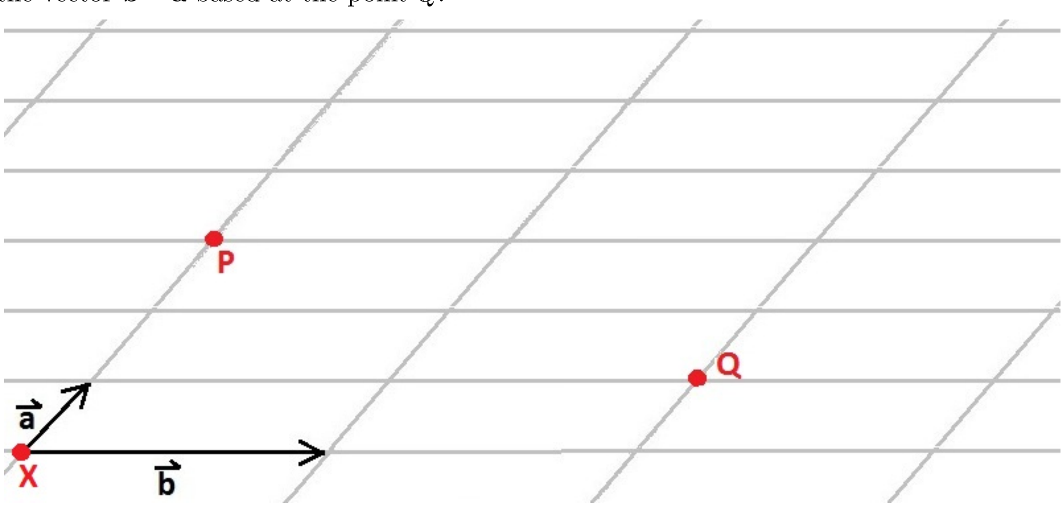
Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(c) Fill in the blanks...

$|\mathbf{a} \times \mathbf{b}|$ computes the ______________________ of the ______________________ spanned by $\mathbf{a}$ and $\mathbf{b}$.

$|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$ computes the ______________________ of the ______________________ spanned by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.

(d) The vectors $\mathbf{a}$ and $\mathbf{b}$ are shown below. They are based a the point X. Sketch the vector $1.5\mathbf{b}$ based at the point P and sketch the vector $\mathbf{b} - \mathbf{a}$ based at the point Q.



2. (9 points) Let ℓ_1 be parametrized by $\mathbf{r}_1(t) = \langle 1, 0, -1\rangle + \langle 1, 2, -1\rangle t$ and ℓ_2 by $\mathbf{r}_2(t) = \langle 1, -4, 1\rangle + \langle -2, -4, 2\rangle t$. Determine if ℓ_1 and ℓ_2 are... (circle the correct answer)

the same, parallel (but not the same), intersecting, or skew.

3. (14 points) Plane old geometry.

(a) Find the (scalar) equation of the plane through the points $A = (2, 2, 1)$, $B = (3, 2, 1)$, and $C = (4, 3, -1)$.

(b) Find the area of the triangle ΔABC (the triangle with vertices A, B, and C).

4. (7 points) Find a formula for the curvature of $y = \sin(x)$.

5. (18 points) Let C be the **right half** of the circle $x^2 + y^2 = 4$.

(a) Parameterize C and find a parameterization for the line tangent to C at $(\sqrt{2}, \sqrt{2})$.

(b) Find the centroid of C.

6. (10 points) Find the curvature of the curve parameterized by $\mathbf{r}(t) = \langle e^t, t^2 + 1, \sin(t) \rangle$.
[Don't try to simplify your answer.]

7. (12 points) Find the TNB-frame for $\mathbf{r}(t) = \langle \cos(t), t, \sin(t) \rangle$.

8. (12 points) No numbers here. Choose **ONE** of the following:

I. Suppose that $\mathbf{a}$ and $\mathbf{b}$ are both orthogonal to $\mathbf{c}$. Show that $2\mathbf{a} - \mathbf{b}$ is orthogonal to $\mathbf{c}$.

II. Prove Lagrange's identity: $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a} \bullet \mathbf{b})^2$.

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1. (18 points) Vector Basics

(a) Find the volume of the parallelepiped spanned by $\mathbf{u} = \langle 2, 0, -1 \rangle$, $\mathbf{v} = \langle 1, 1, 2 \rangle$, and $\mathbf{w} = \langle 1, 2, 1 \rangle$.

(b) Find the angle between $\mathbf{u} = \langle -1, 2, 1 \rangle$ and $\mathbf{w} = \langle 3, 0, 1 \rangle$ (don't worry about evaluating inverse trig. functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(c) Fill in the blanks...

$|\mathbf{a} \times \mathbf{b}|$ computes the ____________________ of the ____________________ spanned by $\mathbf{a}$ and $\mathbf{b}$.

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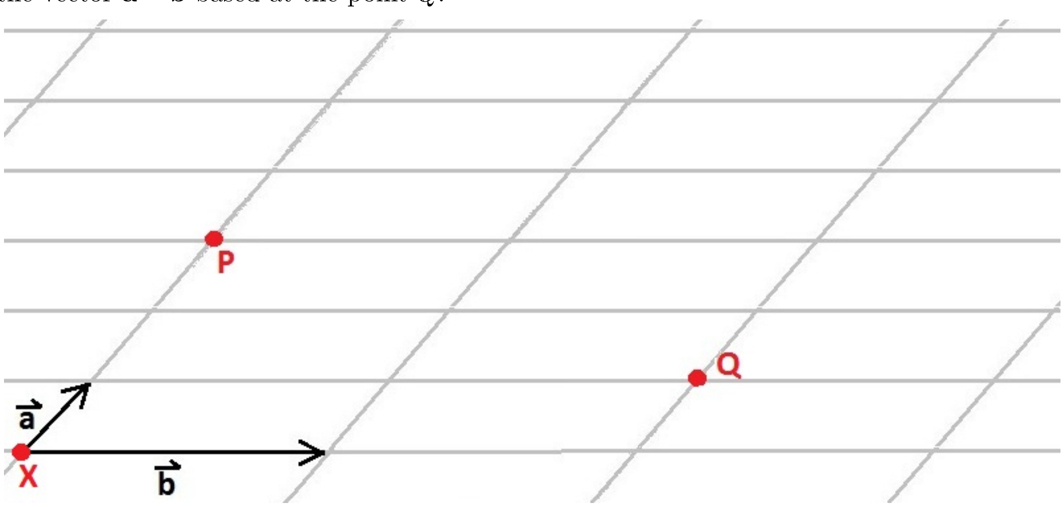
(d) The vectors $\mathbf{a}$ and $\mathbf{b}$ are shown below. They are based a the point X. Sketch the vector $1.5\mathbf{a}$ based at the point P and sketch the vector $\mathbf{a} - \mathbf{b}$ based at the point Q.



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the same, parallel (but not the same), intersecting, or skew.

3. (14 points) Plane old geometry.

(a) Find the (scalar) equation of the plane through the points $A = (1, 2, 2)$, $B = (2, 2, 4)$, and $C = (-1, 3, 3)$.

(b) Find the area of the triangle ΔABC (the triangle with vertices A, B, and C).

4. (7 points) Find a formula for the curvature of $y = x^3$.

5. (18 points) Let C be the **bottom half** of the circle $x^2 + y^2 = 4$.

(a) Parameterize C and find a parameterization for the line tangent to C at $(\sqrt{2}, -\sqrt{2})$.

(b) Find the centroid of C.

6. (10 points) Find the curvature of the curve parameterized by $\mathbf{r}(t) = \langle \cos(t), e^t + 1, t^2 \rangle$.
[Don't try to simplify your answer.]

7. (12 points) Find the TNB-frame for $\mathbf{r}(t) = \langle t, \sin(t), \cos(t) \rangle$.

8. (12 points) No numbers here. Choose **ONE** of the following:

I. Suppose $|\mathbf{r}(t)| = c$ ($\mathbf{r}(t)$ has constant length) for all t. Show that $\mathbf{r}(t)$ is orthogonal to $\mathbf{r}'(t)$.

II. Compute $(\mathbf{a} + \mathbf{b}) \bullet (\mathbf{a} \times \mathbf{b})$. Then give a geometric interpretation of your computation.