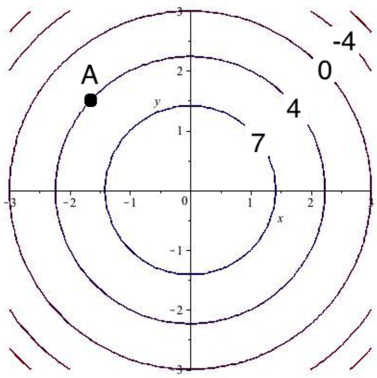
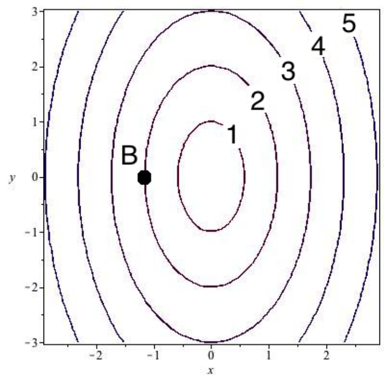
Math 2130-101 **Test #2** **October 16^{th}, 2013**

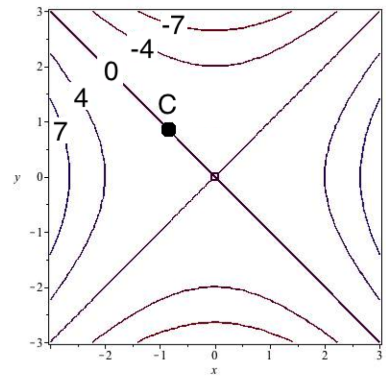
Name: ____________________________________ **Be sure to show your work!**

If $F(x,y)=C$, then $\dfrac{dy}{dx}=-\dfrac{F_x}{F_y}$ If $F(x,y,z)=C$, then $\dfrac{\partial z}{\partial x}=-\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y}=-\dfrac{F_y}{F_z}$

1. (12 points) Three level curve plots are shown below. I have labeled the levels so you know which curves are higher and which are lower.







(a) The plots above correspond to the functions: $f(x,y)=x^2-y^2$, $f(x,y)=9-x^2-y^2$, and $f(x,y)=\sqrt{3x^2+y^2}$. Write the correct formula below each plot.

(b) Sketch a gradient vector at the points A, B, and C. If the vector is $\mathbf{0}$, draw an "X" on the point. [Don't worry about having the correct length. I'm just looking for the correct direction.]

2. (10 points) Let $z=f(u,v)$ where $u=x+y$ and $v=x-y$. Show that $\left(\dfrac{\partial z}{\partial x}\right)\left(\dfrac{\partial z}{\partial y}\right)=\left(\dfrac{\partial z}{\partial u}\right)^2-\left(\dfrac{\partial z}{\partial v}\right)^2$.

3. (8 points) Find an equation for the plane tangent to $x^2y+\sin(xyz)+xz=-2$ at the point $(x,y,z)=(-1,0,2)$.

4. (10 points) Limits

(a) Show the following limit **does** exist: $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2+x^3}{x^2+y^2}$

(b) Show the following limit **does not** exist: $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

5. (10 points) Let $z = \frac{x^2}{y}$. Use a differential (i.e. total derivative) to estimate the percent error in z if x is off by no more than 3% and y is off by no more than 1%.

6. (8 points) Suppose we have a function of two variables: $f(x,y)$. For each question circle **YES** or **NO**. Then briefly explain your answer (in a sentence or two).

(a) Is it possible to have $f_{xy} \neq f_{yx}$? **YES** / **NO**

(b) Suppose f_x and f_y exist everywhere. Can I conclude that f is differentiable? **YES** / **NO**

7. (12 points) Let $f(x, y) = x^3 - xy + y + 1$.

(a) Find the gradient of f and the Hessian matrix of f.

(b) Find the quadratic approximation of f at $(x, y) = (1, 2)$.

(c) Find an classify the critical point(s) of $f(x, y)$.
[Use the "2^{nd}-derivative" test to determine if critical points are relative max's, min's or saddle points.]

8. (10 points) Let $f(x, y) = y^3 + x^2 y + 3x + 4$

(a) Find the directional derivative of f at the point $(x, y) = (1, 0)$ and in the same direction as $\mathbf{v} = \langle -3, 4 \rangle$.

(b) Can the directional derivative of f at the point $(x, y) = (1, 0)$ be equal to 2? **YES** / **NO**

Can it be equal to -10? **YES** / **NO**

Briefly explain your answer.

9. (10 points) Suppose $f(x, y)$ is a "nice" function (with continuous partials of all orders).

(a) $Q(x, y) = 2(x-1) + 3(y-2) + (x-1)^2 + 3(x-1)(y-2) + 3(y-2)^2$ is the quadratic approx. at $(x, y) = (1, 2)$.

$\nabla f(1, 2) = \Big\langle \qquad\qquad\qquad\qquad \Big\rangle \qquad H_f(1, 2) = \Big[\qquad\qquad\qquad\qquad \Big]$

Is $(x, y) = (1, 2)$ a critical point of $f(x, y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

(b) $Q(x, y) = 5 + (-3)(x-3)^2 + 2(x-3)(y+2) + (-3)(y+2)^2$ is the quadratic approx. at $(x, y) = (3, -2)$.

$\nabla f(3, -2) = \Big\langle \qquad\qquad\qquad\qquad \Big\rangle \qquad H_f(3, -2) = \Big[\qquad\qquad\qquad\qquad \Big]$

Is $(x, y) = (3, -2)$ a critical point of $f(x, y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

10. (10 points) Set up but **do not solve** the equations used in the method of Lagrange multipliers for finding the minimum and maximum values of $f(x, y, z) = xy + z^3$ constrained to $x + y^2 + z^3 = 1$. **Circle Your Answer!**

Math 2130-102

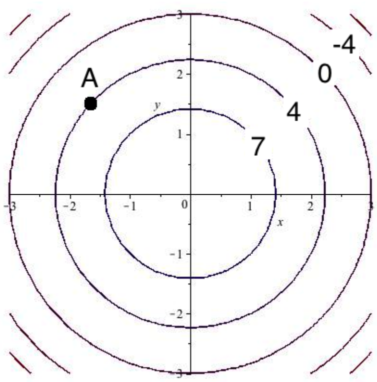
Test #2

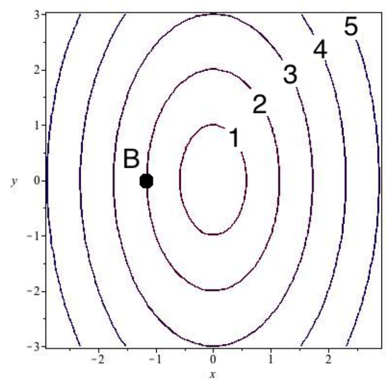
October 16^{th}, 2013

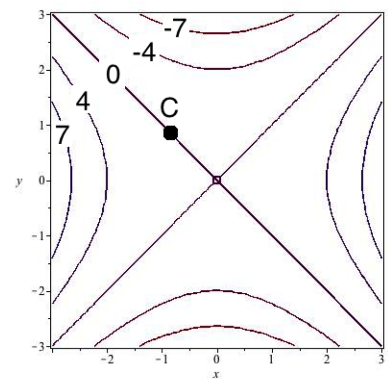
Name: ______________________________________ **Be sure to show your work!**

If $F(x,y) = C$, then $\dfrac{dx}{dy} = -\dfrac{F_x}{F_y}$ If $F(x,y,z) = C$, then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z}$

1. (12 points) Three level curve plots are shown below. I have labeled the levels so you know which curves are higher and which are lower.







(a) The plots above correspond to the functions: $f(x,y) = x^2 - y^2$, $f(x,y) = 9 - x^2 - y^2$, and $f(x,y) = \sqrt{3x^2 + y^2}$. Write the correct formula below each plot.

(b) Sketch a gradient vector at the points A, B, and C. If the vector is **0**, draw an "X" on the point. [Don't worry about having the correct length. I'm just looking for the correct direction.]

2. (8 points) State the **chain rule** for the derivative or partial derivative (whichever makes sense) of w with respect to t where $w = f(x,y,z)$, $x = g(t)$, $y = h(t)$, and $z = \ell(t)$. Make sure you clearly label regular derivatives with d's and partials with ∂'s. If your handwriting leaves this difficult to determine, write "regular" and "partial" and draw arrows to which is which.

3. (10 points) Show the following limit does not exist: $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz + zx}{x^2 + y^2 + z^2}$

4. (10 points) Suppose that $z = xy^3$. I measured and found that $x = 2$ and $y = 1$, but I think my measurement for x might be off by ± 0.2 and my measurement for y might be off by ± 0.1. Use a differential (i.e. total derivative) to estimate the possible error in z.

5. (10 points) Find an equation for the plane tangent to $xe^{xyz} + y^2 z = -1$ at the point $(x, y, z) = (0, 1, -1)$.

6. (8 points) Suppose we have a function of two variables: $f(x, y)$. For each question circle **YES** or **NO**. Then briefly explain your answer (in a sentence or two).

(a) Is it possible to have $f_{xy} \neq f_{yx}$? **YES** / **NO**

(b) Suppose f_x and f_y exist everywhere. Can I conclude that f is differentiable? **YES** / **NO**

7. (12 points) Let $f(x, y) = x^3 + xy + y - 8$.

(a) Find the gradient of f and the Hessian matrix of f.

(b) Find the quadratic approximation of f at $(x, y) = (2, 1)$.

(c) Find an classify the critical point(s) of $f(x, y)$.
[Use the "2^{nd}-derivative" test to determine if critical points are relative max's, min's or saddle points.]

8. (10 points) Let $f(x, y) = xy^3 + 3y + 55$

(a) Find the directional derivative of f at the point $(x, y) = (0, 1)$ and in the same direction as $\mathbf{v} = \langle 4, -3 \rangle$.

(b) Can the directional derivative of f at the point $(x, y) = (0, 1)$ be equal to 10? **YES** / **NO**

Can it be equal to -1? **YES** / **NO**

Briefly explain your answer.

9. (10 points) Suppose $f(x, y)$ is a "nice" function (with continuous partials of all orders).

(a) $Q(x, y) = 2 + 5(x - 3)^2 + 2(x - 3)(y + 2) + 3(y + 2)^2$ is the quadratic approx. at $(x, y) = (3, -2)$.

$\nabla f(3, -2) = \Big\langle \qquad \qquad \Big\rangle$ $\qquad$ $H_f(3, -2) = \Big[\qquad \qquad \Big]$

Is $(x, y) = (3, -2)$ a critical point of $f(x, y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

(b) $Q(x, y) = 6x - 3(y - 4) + x^2 + 2x(y - 4) - 3(y - 4)^2$ is the quadratic approx. at $(x, y) = (0, 4)$.

$\nabla f(0, 4) = \Big\langle \qquad \qquad \Big\rangle$ $\qquad$ $H_f(0, 4) = \Big[\qquad \qquad \Big]$

Is $(x, y) = (0, 4)$ a critical point of $f(x, y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

10. (10 points) Use the method of Lagrange multipliers to find the minimum and maximum **values** of $f(x, y) = 2x + 4y$ constrained to $x^2 + y^2 = 5$.