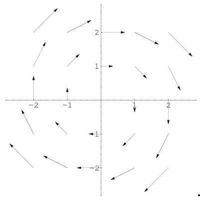
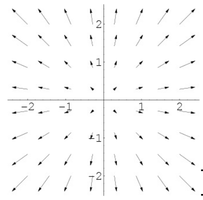
Math 2130-101 **Test #3** **November 12$^{\text{th}}$, 2013**

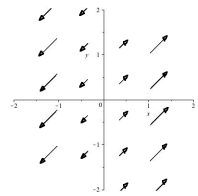
Name: Answer Key **Be sure to show your work!**

1. (18 points) A few vector fields.

(a) The following are plots of several vector fields. Please note that all of the vectors have been scaled down so they do not overlap each other. Write A, B, and C next to the appropriate vector field's formula. Put an X next to the formula whose vector field is **not shown**.


A


B


C

C $\mathbf{F}(x,y) = \left\langle \frac{x}{5}, \frac{x}{5} \right\rangle$	**X** $\mathbf{F}(x,y) = \langle -y, x \rangle$	**X** $\mathbf{F}(x,y) = \langle -y, -x \rangle$	**B** $\mathbf{F}(x,y) = \langle x, y \rangle$
Yes / [No]	Yes / [No]	[Yes] / No	[Yes] / No

For each vector field above, is $\mathbf{F}$ conservative? Circle "Yes" or "No".

Note: I made a mistake when uploading pictures for this test and made this into *sort of* a "trick" question. I originally intended to have A's equation listed as a choice. Oops!

Notice that the vector field $\mathbf{F} = \langle x/5, x/5 \rangle$ does not depend on y. We merely have diagonal vectors which grow (or flip over and grow) as x gets large (or negative). This matches C. Next, $\mathbf{F} = \langle -y, x \rangle$ should have $\mathbf{F}(1,0) = \langle 0, 1 \rangle = \mathbf{j}$ plotted at the point $(x,y) = (1,0)$. This doesn't match A, B, or C. The vector field $\mathbf{F}(x,y) = \langle -y, -x \rangle$ should have $\mathbf{F}(1,1) = \langle -1, -1 \rangle$ plotted at the point $(x,y) = (1,1)$. So we would have a vector pointing from $(1,1)$ to the origin. This doesn't match A, B, or C. Finally, $\mathbf{F}(x,y) = \langle x, y \rangle$ should have vectors pointing radially outward and growing at we move away from the origin. This matches plot B.

Recall that a vector field (defined on all of $\mathbb{R}^2$ with continuous partials) is conservative if $P_y = Q_x$ where $\mathbf{F} = \langle P, Q \rangle$. So we check each formula: For $\mathbf{F} = \langle x/5, x/5$ we get $P_y = 0 \neq 1/5 = Q_x$ (not conservative). For $\mathbf{F} = \langle -y, x \rangle$ we get $P_y = -1 \neq 1 = Q_x$ (not conservative). For $\mathbf{F} = \langle -y, -x \rangle$ we get $P_y = -1 = Q_x$ (conservative). In fact, if $f(x,y) = -xy$, then $\nabla f = \mathbf{F}$. Finally, for $\mathbf{F} = \langle x, y \rangle$ we get $P_y = 0 = Q_x$ (conservative). Again, if $f(x,y) = x^2/2 + y^2/2$, then $\nabla f = \mathbf{F}$.

(b) Compute the divergence and curl of $\mathbf{F}(x,y,z) = \langle yz, xz + 2yz, xy + y^2 + 1 \rangle$. [Show your work!]

$$\text{Div}(\mathbf{F}) = \nabla \bullet \mathbf{F} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \bullet \langle yz, xz + 2yz, xy + y^2 + 1 \rangle = 0 + 2z + 0 = \boxed{2z}$$

$$\text{Curl}(\mathbf{F}) = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz + 2yz & xy + y^2 + 1 \end{vmatrix} = \langle (x + 2y) - (x + 2y), -[y - y], z - z \rangle = \boxed{\langle 0, 0, 0 \rangle}$$

Notice that the curl of $\mathbf{F}$ is zero (also $\mathbf{F}$ has continuous partials on all of $\mathbb{R}^3$). This means $\mathbf{F}$ is a conservative vector field – which is important. If $\mathbf{F}$ wasn't conservative, it wouldn't even have a potential function so the next part of this problem wouldn't make sense!

(c) Find a potential function for the vector field in part (b).

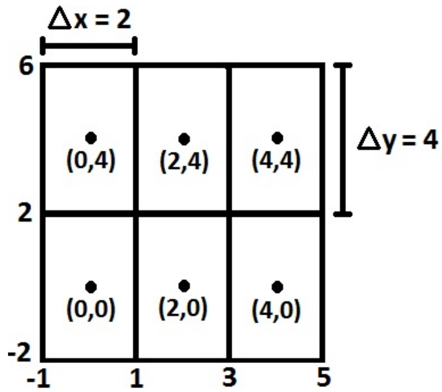
Integrate the first component with respect to x, the second with respect to y, and the third with respect to z...

$$\int yz\,dx = xyz + C_1(y,z) \qquad \int xz + 2yz\,dy = xyz + y^2 z + C_2(x,z) \qquad \int xy + y^2 + 1\,dz = xyz + y^2 z + z + C_3(x,y)$$

Put this together and get that $\boxed{f(x,y,z) = xyz + y^2 z + z} + C$ is a potential function for $\mathbf{F}$ (if C is an arbitrary constant). [In my directions I said to find "a" potential function, not all possible ones, so if you did't include "$+C$", your answer is still correct.

2. (10 points) Use a double Riemann sum to approximate $\iint\limits_R \sqrt{x+y^3}\,dA$ where $R=[-1,5]\times[-2,6]$.

Use midpoint rule and a 3×2 grid of rectangles (3 across and 2 up) to partition R. (Don't worry about simplifying.)

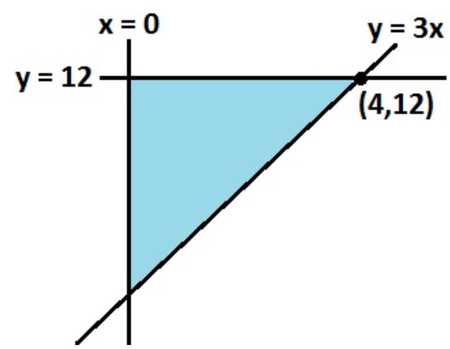


$$\iint_R \sqrt{x+y^3}\,dA \approx$$
$$2\cdot 4\cdot\left(\sqrt{0+0^3}+\sqrt{2+0^3}+\sqrt{4+0^3}+\sqrt{0+4^3}+\sqrt{2+4^3}+\sqrt{4+4^3}\right)$$

3. (14 points) First, sketch the region of integration and then evaluate $\displaystyle\int_0^4\int_{3x}^{12}\frac{e^{2y}}{y}\,dy\,dx$.

Hint: $\int \frac{e^{2y}}{y}\,dy$ cannot be expressed in terms of elementary functions – that is – you can't integrate it.

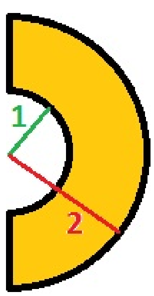
We need to reverse the order of integration to be able to integrate in terms of elementary functions. This region has bounds: $3x\le y\le 12$ and $0\le x\le 4$. After sketching the region we can see that $0\le x\le y/3$ (x's are bounded by the y-axis on the left and the line $y=3x$ on the right). Also, $0\le y\le 12$.



$$\int_0^4\int_{3x}^{12}\frac{e^{2y}}{y}\,dy\,dx=\int_0^{12}\int_0^{y/3}\frac{e^{2y}}{y}\,dx\,dy=\int_0^{12}\frac{e^{2y}}{y}x\Big|_0^{y/3}\,dy=\int_0^{12}\frac{e^{2y}}{y}\cdot\frac{y}{3}\,dy$$

$$=\int_0^{12}\frac{e^{2y}}{3}\,dy=\frac{e^{2y}}{6}\Big|_0^{12}=\boxed{\frac{e^{24}-1}{6}}$$

4. (14 points) Find the centroid of $R=\{(x,y)\,|\,1\le x^2+y^2\le 4 \text{ and } x\ge 0\}$ (the right-half of an annulus centered at the origin). Feel free to use what you know about areas of circles and symmetry to cut down the number of integrals you need to evaluate.



Notice that this region is nothing more than the right-half (i.e. $x\ge 0$) of an annulus (a region between 2 circles). Also, notice that we get $\bar{y}=0$ by symmetry.

Next, $m=\dfrac{\pi\cdot 2^2-\pi\cdot 1^2}{2}=\dfrac{3\pi}{2}$ since the area of an annulus is the difference between the areas of the circles defining it (and don't forget we're only dealing with half).

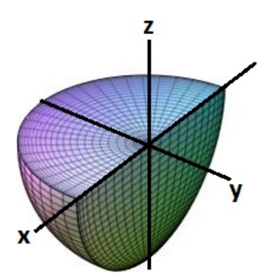
To find $\bar{x}$, we'll need to compute the moment about the y-axis. The double integral defining M_y is best dealt with in terms of polar coordinates where our annular region is described by $1\le r\le 2$ and $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$.

$$M_y=\iint_R x\,dA=\int_{-\pi/2}^{\pi/2}\int_1^2 r\cos(\theta)\cdot r\,dr\,d\theta=\int_{-\pi/2}^{\pi/2}\cos(\theta)\,d\theta\cdot\int_1^2 r^2\,dr=2\cdot\left[\frac{r^3}{3}\right|_1^2=\frac{2}{3}\left(2^3-1^3\right)=\frac{14}{3}$$

Notice that we could "factor" our integral since $r^2\cos(\theta)$ factors into r and θ parts and we have only constant bounds.

Finally, $\bar{x}=\dfrac{M_y}{m}=\dfrac{14/3}{3\pi/2}=\dfrac{28}{9\pi}$. Therefore, $\boxed{(\bar{x},\bar{y})=\left(\dfrac{28}{9\pi},0\right)}$

5. (15 points) Consider the integral: $I=\displaystyle\int_{-4}^4\int_{-\sqrt{16-x^2}}^0\int_{-\sqrt{16-x^2-y^2}}^0 \ln(x^2+y^2+z^2+1)\,dz\,dy\,dx.$



This region is bounded by $-\sqrt{16-x^2-y^2}\le z\le 0$, $-\sqrt{16-x^2}\le y\le 0$, and $-4\le x\le 4$. The first pair of inequalities say that z is bounded between the lower-half of a sphere of radius 4 and the xy-plane. The next inequalities say that y is bounded between a lower-half of a circle and the x-axis. Then x ranges from -4 to 4. Putting this together we get that our region of integration is the left-half of the lower-half of the solid ball of radius 4 (centered at the origin).

When switching to cylindrical coordinates, don't forget that $x^2+y^2=r^2$ and the Jacobian is r. Also, obviously since (x,y) are trapped inside the circle of radius 4 (i.e. $x^2+y^2\le 16$) we get $0\le r\le 4$ and the lower-half of the circle corresponds to $\pi\le\theta\le 2\pi$.

In spherical coordinates, we have that $x^2+y^2+z^2=\rho^2$ and the Jacobian is $\rho^2\sin(\varphi)$. Also, $x^2+y^2+z^2\le 4$ (inside the sphere) translates to $0\le\rho\le 4$ and the lower-half of 3-space corresponds to $\pi/2\le\varphi\le\pi$.

(a) Rewrite I in the following order of integration: $\iiint\quad dy\,dx\,dz$.

Do **not** evaluate the integral.

$$\int_{-4}^{0}\int_{-\sqrt{16-z^2}}^{\sqrt{16-z^2}}\int_{-\sqrt{16-x^2-z^2}}^{0}\ln(x^2+y^2+z^2+1)\,dy\,dx\,dz$$

(b) Rewrite I in terms of cylindrical coordinates.

Do **not** evaluate the integral.

$$\int_{\pi}^{2\pi}\int_{0}^{4}\int_{-\sqrt{16-r^2}}^{0}\ln(r^2+z^2+1)\cdot r\,dz\,dr\,d\theta$$

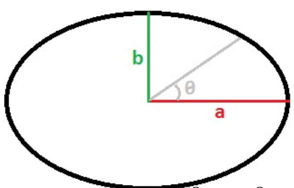
(c) Rewrite I in terms of spherical coordinates.

Do **not** evaluate the integral.

$$\int_{\pi}^{2\pi}\int_{\pi/2}^{\pi}\int_{0}^{4}\ln(\rho^2+1)\cdot\rho^2\sin(\varphi)\,d\rho\,d\varphi\,d\theta$$

6. (14 points) Compute the area inside the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ (assume a and b are some fixed positive real numbers).

Do this by evaluating a double integral and using "modified" polar coordinates. Don't forget the Jacobian!!!



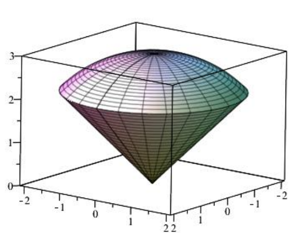
We use "modified" polar coordinates. Specifically, $x=ar\cos(\theta)$ and $y=br\sin(\theta)$. By scaling x and y by a and b, our new x's and y's still satisfy the ellipse's equation: $x^2/a^2+y^2/b^2=1$ (notice when plugging in x and y, the a^2's and b^2's cancel). We need to stay inside the ellipse so $x^2/a^2+y^2/b^2=r^2\cos^2(\theta)+r^2\sin^2(\theta)=r^2\le 1$. Thus $0\le r\le 1$. Next, θ plays essentially the same role it does in polar cooridnates, so $0\le\theta\le 2\pi$ takes us all the way around the ellipse. At this point we just lack the Jacobian.

$$J=\frac{\partial(x,y)}{\partial(r,\theta)}=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial x}{\partial\theta}\\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial\theta}\end{vmatrix}=\begin{vmatrix}a\cos(\theta) & -ar\sin(\theta)\\ b\sin(\theta) & br\cos(\theta)\end{vmatrix}=a\cos(\theta)\cdot br\cos(\theta)-(-ar\sin(\theta)\cdot b\sin(\theta))=abr(\cos^2(\theta)+\sin^2(\theta))=abr$$

Since we assumed $a,b>0$ and since our bounds force $0\le r$, we have $J=abr\ge 0$ (no abs. value needed in the integral).

$$\text{Area}=\iint_R 1\,dA=\int_0^{2\pi}\int_0^1 1\cdot abr\,dr\,d\theta=\int_0^{2\pi}ab\,d\theta\cdot\int_0^1 r\,dr=2\pi\,ab\cdot\left.\frac{r^2}{2}\right|_0^1=\boxed{ab\pi}$$

7. (15 points) Let E be the region bounded below by $z=\sqrt{x^2+y^2}$ and above by $x^2+y^2+z^2=9$.



This solid region is bounded below by a cone and above by the sphere of radius 3. It makes sense to treat this like a z-simple region in which case $\sqrt{x^2+y^2}\le z\le\sqrt{9-x^2-y^2}$. Projecting this down onto the xy-plane, we have a disk. The radius of this disk is determined by the circle where the cone and sphere intersect: $z=\sqrt{x^2+y^2}$ and so $x^2+y^2+z^2=x^2+y^2+(\sqrt{x^2+y^2})^2=9$. Thus $2x^2+2y^2=9$ so that $x^2+y^2=(3/\sqrt{2})^2$. Armed with this it is relatively simple to write the integral in terms of rectangular and cylindrical coordinates.

For spherical coordinates, notice that a ray eminating from the origin first touches the sphere. Therefore, $0\le\rho\le 3$ (the radius of the sphere is 3). Obviously $0\le\theta\le 2\pi$. This only leaves φ. Notice that φ sweeps down from the z-axis (i.e. $\varphi=0$) until we hit the cone. In spherical coordinates the cone's equation is $\rho\cos(\varphi)=\sqrt{x^2+y^2}=r=\rho\sin(\varphi)$ so $\tan(\varphi)=1$. Thus $\varphi=\pi/4$.

(a) Pick an order of integration (for example: $dz\,dy\,dx$) and write $\iiint_E x^2+y^2\,dV$ as an iterated integral in terms of rectangular coordinates. **Do not evaluate this integral!!**

$$\int_{-\frac{3}{\sqrt{2}}}^{\frac{3}{\sqrt{2}}}\int_{-\sqrt{\frac{9}{2}-x^2}}^{\sqrt{\frac{9}{2}-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{9-x^2-y^2}}(x^2+y^2)\,dz\,dy\,dx$$

(b) Write $\iiint_E x^2 + y^2\, dV$ in terms of cylindrical coordinates. Simplify, please. **Do not evaluate this integral!!**

$$\int_0^{2\pi}\int_0^{\frac{3}{\sqrt{2}}}\int_r^{\sqrt{9-r^2}} r^2 \cdot r\, dz\, dr\, d\theta = \int_0^{2\pi}\int_0^{\frac{3}{\sqrt{2}}}\int_r^{\sqrt{9-r^2}} r^3\, dz\, dr\, d\theta$$

(c) Write $\iiint_E x^2 + y^2\, dV$ in terms of spherical coordinates. Simplify, please. **Do not evaluate this integral!!**

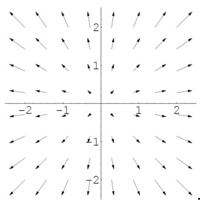
$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{3} \rho^2 \sin^2(\varphi)\cdot \rho^2 \sin(\varphi)\, d\rho\, d\varphi\, d\theta = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{3} \rho^4 \sin^3(\varphi)\, d\rho\, d\varphi\, d\theta$$

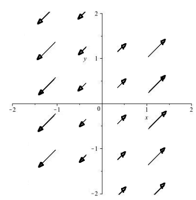
Math 2130-102 **Test #3** **November 12^{th}, 2013**

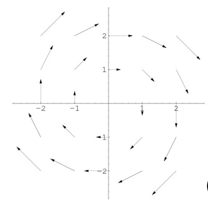
Name: ANSWER KEY **Be sure to show your work!**

1. (18 points) A few vector fields.

(a) The following are plots of several vector fields. Please note that all of the vectors have been scaled down so they do not overlap each other. Write A, B, and C next to the appropriate vector field's formula. Put an X next to the formula whose vector field is **not shown**.


A


B


C

B $\mathbf{F}(x,y) = \left\langle \frac{x}{5}, \frac{x}{5} \right\rangle$	**X** $\mathbf{F}(x,y) = \langle -y, x\rangle$	**X** $\mathbf{F}(x,y) = \langle -y, -x\rangle$	**A** $\mathbf{F}(x,y) = \langle x, y\rangle$
Yes / [No]	Yes / [No]	[Yes] / No	[Yes] / No

For each vector field above, is $\mathbf{F}$ conservative? Circle "Yes" or "No".

This is essentially the same as Section 101's problem #1(a) – just with the plots rearranged.

(b) Compute the divergence and curl of $\mathbf{F}(x,y,z) = \langle yz, xz + 2yz, xy + y^2 + 1\rangle$. [Show your work!]

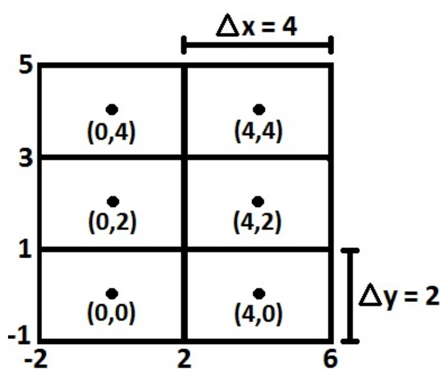
Same as Section 101's problem #1(b).

(c) Find a potential function for the vector field in part (b).

Same as Section 101's problem #1(c).

2. (10 points) Use a double Riemann sum to approximate $\iint_R \sqrt{x^2 + y}\, dA$ where $R = [-2, 6] \times [-1, 5]$.

Use midpoint rule and a 2×3 grid of rectangles (2 across and 3 up) to partition R. (Don't worry about simplifying.)



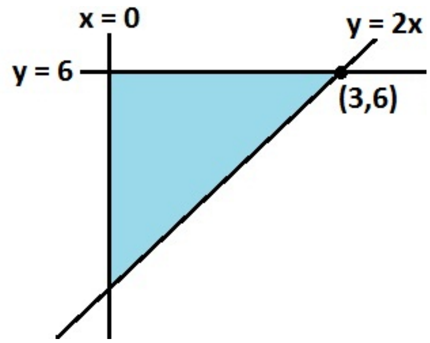
$$\iint_R \sqrt{x^2 + y}\, dA \approx$$

$$4 \cdot 2 \cdot \left(\sqrt{0^2+0} + \sqrt{4^2+0} + \sqrt{0^2+2} + \sqrt{4^2+2} + \sqrt{0^2+4} + \sqrt{4^2+4}\right)$$

3. (14 points) First, sketch the region of integration and then evaluate $\displaystyle\int_0^3 \int_{2x}^6 \frac{\cos(y)}{y}\,dy\,dx$.

Hint: $\int \frac{\cos(y)}{y}\,dy$ cannot be expressed in terms of elementary functions – that is – you can't integrate it.

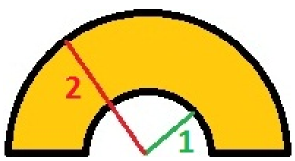
We need to reverse the order of integration to be able to integrate in terms of elementary functions. This region has bounds: $2x \le y \le 6$ and $0 \le x \le 3$. After sketching the region we can see that $0 \le x \le y/2$ (x's are bounded by the y-axis on the left and the line $y = 2x$ on the right). Also, $0 \le y \le 6$.



$$\int_0^3 \int_{2x}^6 \frac{\cos(y)}{y}\,dy\,dx = \int_0^6 \int_0^{y/2} \frac{\cos(y)}{y}\,dx\,dy = \int_0^6 \frac{\cos(y)}{y} x\Big|_0^{y/2}\,dy = \int_0^6 \frac{\cos(y)}{y}\cdot\frac{y}{2}\,dy$$

$$\int_0^6 \frac{\cos(y)}{2}\,dy = \frac{\sin(y)}{2}\Big|_0^6 = \frac{1}{2}\Big(\sin(6)-\sin(0)\Big) = \boxed{\frac{\sin(6)}{2}}$$

4. (14 points) Find the centroid of $R = \{(x,y)\,|\,1 \le x^2+y^2 \le 4 \text{ and } y \ge 0\}$ (the upper-half of an annulus centered at the origin). Feel free to use what you know about areas of circles and symmetry to cut down the number of integrals you need to evaluate.



Notice that this region is nothing more than the upper-half (i.e. $y \ge 0$) of an annulus (a region between 2 circles). Also, notice that we get $\bar{x} = 0$ by symmetry.

Next, $m = \dfrac{\pi\cdot 2^2 - \pi\cdot 1^2}{2} = \dfrac{3\pi}{2}$ since the area of an annulus is the difference between the areas of the circles defining it (and don't forget we're only dealing with half).

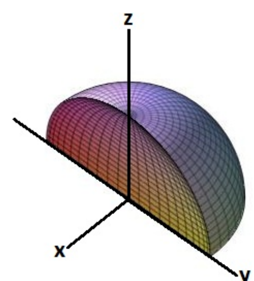
To find $\bar{y}$, we'll need to compute the moment about the x-axis. The double integral defining M_x is best dealt with in terms of polar coordinates where our annular region is described by $1 \le r \le 2$ and $0 \le \theta \le \pi$.

$$M_x = \iint_R y\,dA = \int_0^\pi \int_1^2 r\sin(\theta)\cdot r\,dr\,d\theta = \int_0^\pi \sin(\theta)\,d\theta \cdot \int_1^2 r^2\,dr = 2\cdot\left[\frac{r^3}{3}\right|_1^2 = \frac{2}{3}\Big(2^3-1^3\Big) = \frac{14}{3}$$

Notice that we could "factor" our integral since $r^2\sin(\theta)$ factors into r and θ parts and we have only constant bounds.

Finally, $\bar{y} = \dfrac{M_x}{m} = \dfrac{14/3}{3\pi/2} = \dfrac{28}{9\pi}$. Therefore, $\boxed{(\bar{x},\bar{y}) = \left(0, \dfrac{28}{9\pi}\right)}$

5. (15 points) Consider the integral: $I = \displaystyle\int_{-4}^0 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_0^{\sqrt{16-x^2-y^2}} \sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$.



This region is bounded by $0 \le z \le \sqrt{16-x^2-y^2}$, $-\sqrt{16-x^2} \le y \le \sqrt{16-x^2}$, and $-4 \le x \le 0$. The first pair of inequalities say that z is bounded between the xy-plane and the upper-half of a sphere of radius 4. The next inequalities say that y is bounded between by the circle and of radius 4. Then x ranges from -4 to 0, so we only have the left-half of the circle. Putting this together we get that our region of integration is the back-half of the upper-half of the solid ball of radius 4 (centered at the origin).

When switching to cylindrical coordinates, don't forget that $x^2+y^2 = r^2$ and the Jacobian is r. Also, obviously since (x,y) are trapped inside the circle of radius 4 (i.e. $x^2+y^2 \le 16$) we get $0 \le r \le 4$ and the left-half of the circle corresponds to $\pi/2 \le \theta \le 3\pi/2$.

In spherical coordinates, we have that $x^2+y^2+z^2 = \rho^2$ and the Jacobian is $\rho^2\sin(\varphi)$. Also, $x^2+y^2+z^2 \le 4$ (inside the sphere) translates to $0 \le \rho \le 4$ and the upper-half of 3-space corresponds to $0 \le \varphi \le \pi/2$.

(a) Rewrite I in the following order of integration: $\iiint\ \ dy\,dx\,dz$.

Do **not** evaluate the integral.

$$\int_0^4 \int_{-\sqrt{16-z^2}}^0 \int_{-\sqrt{16-x^2-z^2}}^{\sqrt{16-x^2-z^2}} \sqrt{x^2+y^2+z^2}\,dy\,dx\,dz$$

(b) Rewrite I in terms of cylindrical coordinates.

Do **not** evaluate the integral.

$$\int_{\pi/2}^{3\pi/2} \int_0^4 \int_0^{\sqrt{16-r^2}} \sqrt{r^2+z^2}\cdot r\,dz\,dr\,d\theta$$

(c) Rewrite I in terms of spherical coordinates.

Do **not** evaluate the integral.

$$\int_{\pi/2}^{3\pi/2} \int_0^{\pi/2} \int_0^4 \rho \cdot \rho^2 \sin(\varphi)\, d\rho\, d\varphi\, d\theta$$

6. (14 points) Compute the area inside the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (assume a and b are some fixed positive real numbers).

Do this by evaluating a double integral and using "modified" polar coordinates. Don't forget the Jacobian!!!

This is the same as Section 101's problem #6.

7. (15 points) Let E be the region bounded below by $z = \sqrt{x^2 + y^2}$ and above by $x^2 + y^2 + z^2 = 4$.

This problem is almost the same as Section 101's problem #7, so I'll just provide the answers.

(a) Pick an order of integration (for example: $dz\, dy\, dx$) and write $\iiint_E x^2 + y^2\, dV$ as an iterated integral in terms of rectangular coordinates. **Do not evaluate this integral!!**

$$\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} (x^2 + y^2)\, dz\, dy\, dx$$

(b) Write $\iiint_E x^2 + y^2\, dV$ in terms of cylindrical coordinates. Simplify, please. **Do not evaluate this integral!!**

$$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} r^2 \cdot r\, dz\, dr\, d\theta = \int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} r^3\, dz\, dr\, d\theta$$

(c) Write $\iiint_E x^2 + y^2\, dV$ in terms of spherical coordinates. Simplify, please. **Do not evaluate this integral!!**

$$\int_0^{2\pi} \int_0^{\pi/4} \int_0^2 \rho^2 \sin^2(\varphi) \cdot \rho^2 \sin(\varphi)\, d\rho\, d\varphi\, d\theta = \int_0^{2\pi} \int_0^{\pi/4} \int_0^2 \rho^4 \sin^3(\varphi)\, d\rho\, d\varphi\, d\theta$$