Math 2130-101

Test #3

November 12^{th}, 2013

Name: ______________________________ **Be sure to show your work!**

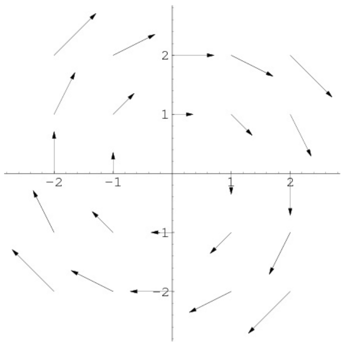
$$\begin{aligned} x &= \rho\cos(\theta)\sin(\varphi) \\ y &= \rho\sin(\theta)\sin(\varphi) \\ z &= \rho\cos(\varphi) \end{aligned}$$

$$J = \rho^2\sin(\varphi)$$

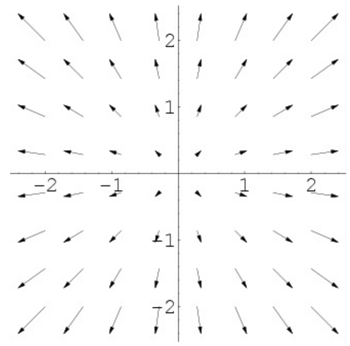
$$\cos^2(\theta) = \tfrac{1}{2}\left(1+\cos(2\theta)\right)$$

1. (18 points) A few vector fields.

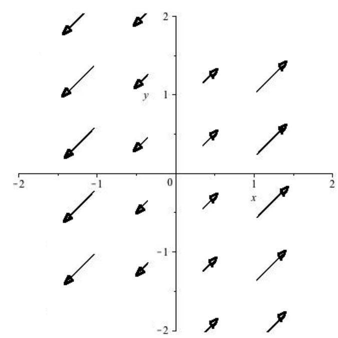
(a) The following are plots of several vector fields. Please note that all of the vectors have been scaled down so they do not overlap each other. Write A, B, and C next to the appropriate vector field's formula. Put an X next to the formula whose vector field is **not shown**.



A



B



C

☐ $\mathbf{F}(x,y) = \left\langle \dfrac{x}{5}, \dfrac{x}{5} \right\rangle$ Yes / No

☐ $\mathbf{F}(x,y) = \langle -y, x\rangle$ Yes / No

☐ $\mathbf{F}(x,y) = \langle -y, -x\rangle$ Yes / No

☐ $\mathbf{F}(x,y) = \langle x, y\rangle$ Yes / No

For each vector field above, is **F** conservative? Circle "Yes" or "No".

(b) Compute the divergence and curl of $\mathbf{F}(x,y,z) = \langle yz, xz+2yz, xy+y^2+1\rangle$. [Show your work!]

(c) Find a potential function for the vector field in part (b).

2. (10 points) Use a double Riemann sum to approximate $\iint_R \sqrt{x+y^3}\,dA$ where $R = [-1,5] \times [-2,6]$.

Use midpoint rule and a 3×2 grid of rectangles (3 across and 2 up) to partition R. (Don't worry about simplifying.)

3. (14 points) First, sketch the region of integration and then evaluate $\displaystyle\int_0^4 \int_{3x}^{12} \frac{e^{2y}}{y}\,dy\,dx$.

Hint: $\int \frac{e^{2y}}{y}\,dy$ cannot be expressed in terms of elementary functions – that is – you can't integrate it.

4. (14 points) Find the centroid of $R = \{(x, y) \,|\, 1 \leq x^2 + y^2 \leq 4 \text{ and } x \geq 0\}$ (the right-half of an annulus centered at the origin). Feel free to use what you know about areas of circles and symmetry to cut down the number of integrals you need to evaluate.

$$m = \iint_R 1\, dA \qquad M_y = M_{x=0} = \iint_R x\, dA \qquad M_x = M_{y=0} = \iint_R y\, dA$$

5. (15 points) Consider the integral: $I = \displaystyle\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{0} \int_{-\sqrt{16-x^2-y^2}}^{0} \ln(x^2 + y^2 + z^2 + 1)\, dz\, dy\, dx.$

(a) Rewrite I in the following order of integration: $\iiint \quad dy\, dx\, dz$.

Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.

Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.

Do **not** evaluate the integral.

6. (14 points) Compute the area inside the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (assume a and b are some fixed positive real numbers). Do this by evaluating a double integral and using "modified" polar coordinates. Don't forget the Jacobian!!!

7. (15 points) Let E be the region bounded below by $z = \sqrt{x^2 + y^2}$ and above by $x^2 + y^2 + z^2 = 9$.

(a) Pick an order of integration (for example: $dz\,dy\,dx$) and write $\iiint_E x^2 + y^2\,dV$ as an iterated integral in terms of rectangular coordinates. **Do not evaluate this integral!!**

(b) Write $\iiint_E x^2 + y^2\,dV$ in terms of cylindrical coordinates. Simplify, please. **Do not evaluate this integral!!**

(c) Write $\iiint_E x^2 + y^2\,dV$ in terms of spherical coordinates. Simplify, please. **Do not evaluate this integral!!**

Math 2130-102 # Test #3 **November 12^{th}, 2013**

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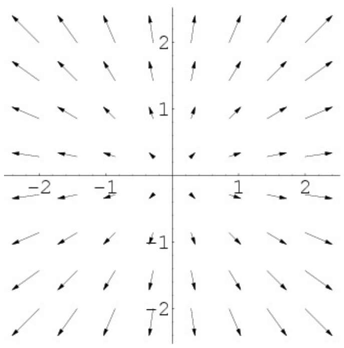
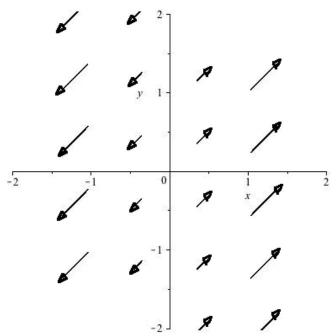
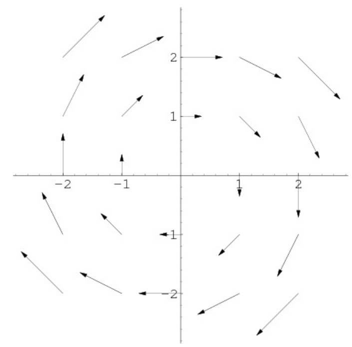


A



B



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For each vector field above, is $\mathbf{F}$ conservative? Circle "Yes" or "No".

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Use midpoint rule and a 2×3 grid of rectangles (2 across and 3 up) to partition R. (Don't worry about simplifying.)

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Hint: $\int \frac{\cos(y)}{y}\,dy$ cannot be expressed in terms of elementary functions – that is – you can't integrate it.

4. (14 points) Find the centroid of $R = \{(x, y) \,|\, 1 \le x^2 + y^2 \le 4 \text{ and } y \ge 0\}$ (the upper-half of an annulus centered at the origin). Feel free to use what you know about areas of circles and symmetry to cut down the number of integrals you need to evaluate.

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