Math 2130 # Final Exam **December 11^{th}, 2014**

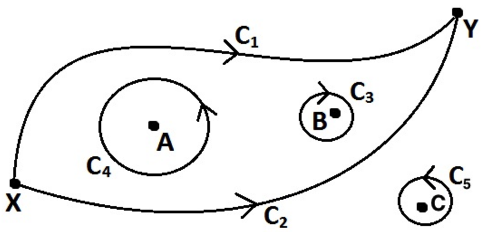
Name: ______________________________________ **Be sure to show your work!**

1. (15 points) Let $\mathbf{F}(x, y, z) = \langle yz + 2xy, x^2 + xz, xy + 1 \rangle$ and let C be the curve parameterized by $\mathbf{r}(t) = \langle 2\cos(t), t, 2\sin(t) \rangle$ where $0 \le t \le 2\pi$.

(a) Verify that $\mathbf{F}$ is conservative and find a potential function.

(b) Set up the line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$ using the given parameterization for C.

(c) Compute $\int_C \mathbf{F} \bullet d\mathbf{r}$.



$\Longleftarrow$ Both C_1 and C_2 begin at X and end at Y.

2. (6 points) Let $\mathbf{F}(x,y) = \langle P(x,y), Q(x,y)\rangle$ be a vector field such that P and Q have continuous first partials and in addition, $P_y = Q_x$ everywhere except at the points A, B, and C. Suppose that $\int_{C_2} \mathbf{F} \bullet d\mathbf{r} = 3$, $\int_{C_3} \mathbf{F} \bullet d\mathbf{r} = 8$, $\int_{C_4} \mathbf{F} \bullet d\mathbf{r} = 2$, and $\int_{C_5} \mathbf{F} \bullet d\mathbf{r} = 10$.

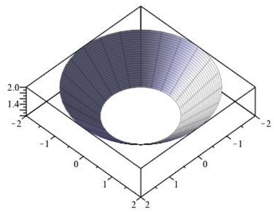
$$\text{Then } \int_{C_1} P(x,y)\,dx + Q(x,y)\,dy \quad = \quad \underline{\hspace{6cm}}.$$

3. (10 points) Compute the area inside the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ using a line integral.

4. (10 points) Let C be the boundary of the square with vertices $(1,2)$, $(3,2)$, $(3,4)$, and $(1,4)$ oriented counter-clockwise. Compute $\displaystyle\int_C \left(\frac{\sin(x)}{x} + y^2\right) dx + \left(3x + \cos(\sqrt{y^4+1})\right) dy$.

5. (14 points) Find the centroid of the part of the cone $z = \sqrt{x^2 + y^2}$ which lies between $z = 1$ and $z = 2$.

Note: This is a **surface**. You should be computing **surface integrals**.



$$m = \iint_S dS \qquad M_{yz} = \iint_S x\, dS \qquad M_{xz} = \iint_S y\, dS \qquad M_{xy} = \iint_S z\, dS$$

6. (14 points) The **divergence theorem** might be helpful.

(a) Let S_1 be the upper hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$. Let S_2 be the disk $x^2 + y^2 \leq 4$ in the xy-plane. Orient both S_1 and S_2 upward. Suppose that $\mathbf{F}$ is a smooth vector field such that $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S} = 10$ and $\nabla \bullet \mathbf{F} = 3$. Find $\iint_{S_2} \mathbf{F} \bullet d\mathbf{S}$.

(b) Compute the flux integral $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$ where S_1 is the unit sphere (i.e. $x^2+y^2+z^2 = 1$) oriented outward and $\mathbf{F}(x, y, z) = \left\langle x^3 + \sqrt{y^{10} + z^{10}}, e^{xz} + y^3, \sin(x^{15} + y + 1) + z^3 \right\rangle$.

7. (15 points) Let S_1 be the surface parameterized by $\mathbf{r}(u, v) = \langle u\sin(v), u\cos(v), v^2\rangle$ where $0 \leq u \leq 3$ and $-\pi \leq v \leq 0$.

(a) Find both orientations for S_1.

(b) Set up but **do not evaluate** the surface integral $\iint_{S_1} x^3 e^y \cos(z)\, dS$.

(c) Set up but **do not evaluate** the flux integral $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$ where S_1 is oriented upward and $\mathbf{F}(x, y, z) = \langle x^2 + y^2, z, 5\rangle$.

8. (16 points) Let S_1 be the part of the plane $2x+y+z=2$ lying in the first octant and oriented upward. Verify Stokes' Theorem for the surface S_1, its boundary, and the vector field $\mathbf{F}(x,y,z) = \langle y, x, yz \rangle$.