Math 2130-101 # Test #1 **September 16^{th}, 2014**

Name: ________________________________ **Be sure to show your work!**

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \qquad \mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t) \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$m = \int_C \rho\, ds \qquad (\bar{x}, \bar{y}, \bar{z}) = \frac{1}{m}\left(\int_C x\rho\, ds, \int_C y\rho\, ds, \int_C z\rho\, ds\right) \qquad \kappa = \frac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{\frac{3}{2}}}$$

1. (14 points) Vector Basics: Let $\mathbf{v} = \langle -1, 3, 1\rangle$ and $\mathbf{w} = \langle 1, 1, 2\rangle$.

(a) Find a **unit** vector which is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$.

(b) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trig. functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(c) Match each expression with a corresponding statement describing what is being computed...

☐	$\mathbf{a} \bullet \mathbf{b} = 0$	**A)**	$\pm$ the volume of a parallelepiped
☐	$\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c})$	**B)**	nonsense
☐	$\lvert\mathbf{a} \times \mathbf{b}\rvert$	**C)**	the vectors are orthogonal
☐	$(\mathbf{a} \bullet \mathbf{b}) \times (\mathbf{b} \bullet \mathbf{c})$	**D)**	the area of a parallelogram

2. (12 points) Let ℓ_1 be parametrized by $\mathbf{r}_1(t) = \langle 1, 2, 0\rangle + \langle -1, 3, 1\rangle t$ and let ℓ_2 be the line which passes through the points $P = (0, 5, 1)$ and $Q = (2, -1, -1)$. Determine if ℓ_1 and ℓ_2 are... (circle the correct answer)

the same, parallel (but not the same), intersecting, or skew.

3. (14 points) A few points...

(a) Find the (scalar) equation of the plane through the points $A = (2, 1, 0)$, $B = (3, 2, 1)$, and $C = (4, 1, -1)$.

(b) Find the area of the triangle with vertices $A = (2, 1, 0)$, $B = (3, 2, 1)$, and $C = (4, 1, -1)$ (these are the same points as in part (a)).

(c) Find a plane which is perpendicular to the plane $x + 2y + 3z + 4 = 0$ and contains the points $P = (1, 1, 1)$ and $Q = (2, 0, 1)$.

4. (10 points) Parameterize the ellipse $\frac{(x-1)^2}{3^2} + \frac{(y-4)^2}{5^2} = 1$. Then set up (but do **not** evaluate) an integral which computes its arc length.

5. (16 points) Let C be the helix parameterized by $\mathbf{r}(t) = \langle 3\sin(t), 4t, 3\cos(t)\rangle$, $-\pi \leq t \leq \pi$.

(a) Compute the **TNB**-frame for C.

(b) Find the curvature of C.

(c) Set up (but do **not** evaluate) the line integral $\int_C (x^2+z^2)e^y\,ds$ [Please simplify your answer.]

(d) Circle the correct answer: C IS / IS NOT a planar curve.

6. (10 points) Suppose that a particle has a constant acceleration vector $\mathbf{a}(t) = -2\mathbf{j}$. Its initial velocity vector was $\mathbf{v}_0 = \mathbf{i} + 5\mathbf{j}$ and its initial position was $\mathbf{r}_0 = 10\mathbf{j}$. Find a formula for the position of this particle, $\mathbf{r}(t)$, at time t (assume $\mathbf{r}(t)$ is measured in meters and t in seconds).

What was the particle's initial **speed**? ____________

7. (14 points) Let C be parameterized by $\mathbf{r}(t) = \langle t, e^t, \sin(t) \rangle$ where $-\pi \leq t \leq \pi$.

(a) Compute the curvature of C.

(b) Find the tangential and normal components of acceleration.

$a_T =$ ______________________ $a_N =$ ______________________

(c) Set up (but do **not** evaluate) the line integral $\int_C (x^2 y + z)\, ds$

8. (10 points) No numbers here. Choose **ONE** of the following:

I. Derive the special formula for curvature of a graph of a function $y = f(x)$ from the curvature formula (use the one with a cross product in it).

II. Let $\mathbf{a}$ and $\mathbf{b}$ be any two vectors. Simplify $(2\mathbf{a} - 3\mathbf{b}) \bullet (\mathbf{a} \times \mathbf{b})$. What does this mean geometrically?

Math 2130-103 — **Test #1** — **September 16^{th}, 2014**

Name: ____________________________________ **Be sure to show your work!**

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \qquad \mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t) \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$m = \int_C \rho\, ds \qquad (\bar{x}, \bar{y}, \bar{z}) = \frac{1}{m}\left(\int_C x\rho\, ds, \int_C y\rho\, ds, \int_C z\rho\, ds\right) \qquad \kappa = \frac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{\frac{3}{2}}}$$

1. (14 points) Vector Basics: Let $\mathbf{v} = \langle -1, -3, 1\rangle$ and $\mathbf{w} = \langle 1, 1, 2\rangle$.

(a) Find the vector of *length* 10 which points in the direction *opposite* that of $\mathbf{v}$.

(b) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trig. functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(c) Match each expression with a corresponding statement describing what is being computed...

☐	$\mathbf{a} \bullet \mathbf{b} = 0$	**A)** the area of a parallelogram
☐	$\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c})$	**B)** the vectors are orthogonal
☐	$\lvert\mathbf{a} \times \mathbf{b}\rvert$	**C)** nonsense
☐	$(\mathbf{a} \bullet \mathbf{b}) \times (\mathbf{b} \bullet \mathbf{c})$	**D)** $\pm$ the volume of a parallelepiped

2. (12 points) Let ℓ_1 be parametrized by $\mathbf{r}_1(t) = \langle 1, 2, 0\rangle + \langle -1, 3, 1\rangle t$ and let ℓ_2 be the line which passes through the points $P = (-2, 1, -3)$ and $Q = (-1, 3, -1)$. Determine if ℓ_1 and ℓ_2 are... (circle the correct answer)

the same, parallel (but not the same), intersecting, or skew.

3. (14 points) A few points...

(a) Find the (scalar) equation of the plane through the points $A = (1, 0, -1)$, $B = (3, 2, 0)$, and $C = (2, 3, 1)$.

(b) Parameterize the line which is perpendicular to the plane $-5x + y + 2z = 13$ and which passes through the point $P = (0, 2, -1)$.

(c) Is the line found in part (b) orthogonal, parallel, both orthogonal and parallel, or neither orthogonal nor parallel to the plane found in part (a)? Explain.

4. (10 points) Let $y = x^3 + 1$ where $-1 \leq x \leq 2$. Parameterize the graph of this function. Also, find its curvature.

5. (16 points) Let C be the helix parameterized by $\mathbf{r}(t) = \langle 3\cos(t), 5\sin(t), 4\cos(t)\rangle$, $-\pi \le t \le \pi$.

(a) Compute the **TNB**-frame for C.

(b) Find the curvature of C.

(c) Set up (but do **not** evaluate) the line integral $\int_C (x^2 + z^2)e^y \, ds$ [Please simplify your answer.]

(d) Circle the correct answer: C IS / IS NOT a planar curve.

6. (10 points) Suppose that a particle has a constant acceleration vector $\mathbf{a}(t) = -4\mathbf{j}$. Its initial velocity vector was $\mathbf{v}_0 = 2\mathbf{i} - \mathbf{j}$ and its initial position was $\mathbf{r}_0 = 100\mathbf{j}$. Find a formula for the position of this particle, $\mathbf{r}(t)$, at time t (assume $\mathbf{r}(t)$ is measured in meters and t in seconds).

What was the particle's initial **speed**? ______________

7. (14 points) Let C be parameterized by $\mathbf{r}(t) = \langle t, t^3, e^t \rangle$ where $0 \leq t \leq 6\pi$.

(a) Compute the curvature of C.

(b) Find the tangential and normal components of acceleration.

$a_T =$ ________________________ $a_N =$ ________________________

(c) Set up (but do **not** evaluate) the line integral $\int_C (x^2y + z)\, ds$

8. (10 points) No numbers here. Choose **ONE** of the following:

I. Compute and simplify $\dfrac{d}{dt}\left[\mathbf{r} \bullet (\mathbf{r}' \times \mathbf{r}'')\right]$ where $\mathbf{r}(t)$ is a smooth vector valued function.

II. It is true that $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}|$? Either explain why this always holds or explain what would make it hold or fail.