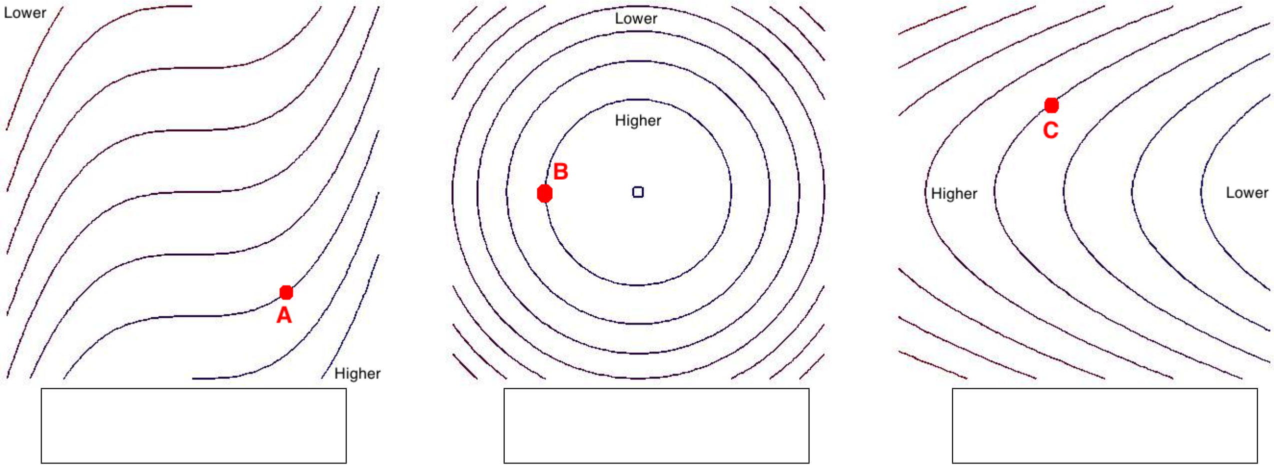
Math 2130-101 & 103 **Test #2** **October 15^{th}, 2014**

Name: ____________________________________ **Be sure to show your work!**

If $F(x,y)=C$, then $\dfrac{dy}{dx}=-\dfrac{F_x}{F_y}$ If $F(x,y,z)=C$, then $\dfrac{\partial z}{\partial x}=-\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y}=-\dfrac{F_y}{F_z}$

1. (11 points) Three level curve plots are shown below. I have labeled the levels so you know which curves are higher and which are lower.



(a) The plots above correspond to 3 of the functions listed here: $f(x,y)=9-x^2-y^2$, $f(x,y)=\sqrt{x^2+y^2}$, $f(x,y)=3x^2-4y$, $f(x,y)=3x-4y^2$, and $f(x,y)=x^3-y$. Write the correct formula below each plot.

(b) Sketch a gradient vector at the points A, B, and C. If the vector is $\mathbf{0}$, draw an "X" on the point. [Don't worry about having the correct length. I'm just looking for the correct direction.]

2. (8 points) Suppose we have a function of two variables: $f(x,y)$. Create a diagram showing how the following statements are related:

(1) f has continuous first partials

(2) the first partials of f exist

(3) f is continuous

(4) f is differentiable

For example: "(1) $\Longleftarrow$ (2) $\Longleftrightarrow$ (3) $\Longrightarrow$ (4)" is a wrong answer.

3. (8 points) Let $z=f(x,y)$, $x=g(u,v)$, $y=h(u,v)$, $u=m(t)$, and $v=n(t)$. State the chain rule for the derivative of z with respect to t. Make sure you clearly indicate which derivatives are partial derivatives and which are regular derivatives. [You may want to draw a variable "tree" first.]

4. (13 points) Let $x + \sin(xy) + e^{yz} + y^2 z = 2$ and let $F(x, y, z) = x + \sin(xy) + e^{yz} + y^2 z$

(a) Compute the directional derivative of $F(x, y, z)$ at $(x, y, z) = (1, 0, 2)$ in the direction of the vector $\mathbf{v} = \langle 2, 1, -2 \rangle$.

(b) Find an equation for the plane tangent to the above surface at the point $(x, y, z) = (1, 0, 2)$.

(c) Considering z as a variable depending on x and y (defined implicitly above), find $\dfrac{\partial z}{\partial x}$.

5. (10 points) Limits

(a) Show the following limit **does** exist: $\displaystyle \lim_{(x,y)\to(0,0)} \frac{5x^3 + y^4}{x^2 + y^2}$

(b) Show the following limit **does not** exist: $\displaystyle \lim_{(x,y,z)\to(0,0,0)} \frac{x + y + z}{x^2 + y^2 + z^2}$

6. (14 points) Let $f(x, y) = x^2y - xy$.

(a) Find the gradient of f and the Hessian matrix of f.

(b) Find the quadratic approximation of f at $(x, y) = (2, -1)$.

(c) Find an classify the critical point(s) of $f(x, y)$.
[Use the "2^{nd}-derivative" test to determine if critical points are relative max's, min's or saddle points.]

7. (10 points) Assume that the function $g(x, y)$ is differentiable.

(a) Suppose that $\nabla g(3, 1) = \langle -1, 4\rangle$. What is the maximum possible value of $D_{\mathbf{u}}g(3, 1)$? Give a unit vector which causes this maximum to occur.

(b) Again, suppose $\nabla g(3, 1) = \langle -1, 4\rangle$. Is is possible to have $D_{\mathbf{u}}g(3, 1) = -3$? Why or why not?

8. (14 points) Suppose $f(x,y)$ is a "nice" function (with continuous partials of all orders).

(a) $Q(x,y) = 1 - x^2 + x(y-4) - 3(y-4)^2$ is the quadratic approx. at $(x,y) = (0,4)$.

$\nabla f(0,4) = \Big\langle \qquad \Big\rangle \qquad H_f(0,4) = \Big[\qquad \Big]$

Is $(x,y) = (0,4)$ a critical point of $f(x,y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

(b) $Q(x,y) = 4(x-1) + 5(x-1)^2 + 3(x-1)(y+2) - 2(y+2)^2$ is the quadratic approx. at $(x,y) = (1,-2)$.

$\nabla f(1,-2) = \Big\langle \qquad \Big\rangle \qquad H_f(1,-2) = \Big[\qquad \Big]$

Is $(x,y) = (1,-2)$ a critical point of $f(x,y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

9. (12 points) Use the method of Lagrange multipliers to find the minimum and maximum values of $f(x,y) = x^2 y$ constrained to $x^2 + y^2 = 6$. [Carefully show all of your work.]