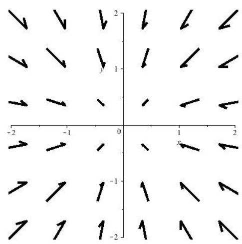
Name: ______________________________ **Be sure to show your work!**

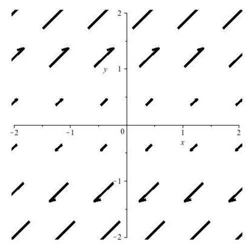
$$x = \rho\cos(\theta)\sin(\varphi) \qquad J = \rho^2\sin(\varphi)$$
$$y = \rho\sin(\theta)\sin(\varphi)$$
$$z = \rho\cos(\varphi) \qquad \cos^2(\theta) = \tfrac{1}{2}(1+\cos(2\theta))$$

1. (15 points) A few vector fields.

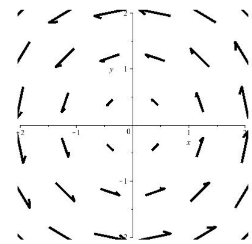
(a) The following are plots of several vector fields. Please note that all of the vectors have been scaled down so they do not overlap each other. Write A, B, and C next to the appropriate vector field's formula. Put an X next to the formula whose vector field is **not shown**.



A



B



C

$\mathbf{F}(x,y) = \langle -y, x\rangle$	$\mathbf{F}(x,y) = \langle -x, -y\rangle$	$\mathbf{F}(x,y) = \langle y, x\rangle$	$\mathbf{F}(x,y) = \langle y, y\rangle$
Yes / No	Yes / No	Yes / No	Yes / No

For each vector field above, is **F** conservative? Circle "Yes" or "No".

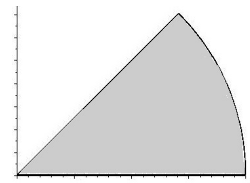
(b) Compute the divergence and curl of $\mathbf{F}(x,y,z) = \langle x^3y^2z, xz^2+y, z^5\rangle$. [Show your work!]

(c) Find a potential function for $\mathbf{F}(x,y,z) = \langle 2x+yz, 2yz+xz, y^2+xy+1\rangle$.

2. (10 points) Use a double Riemann sum to approximate $\displaystyle\iint_R \ln(x+y^2)\,dA$ where $R = [2,8]\times[1,3]$.

Use midpoint rule and a 2×2 grid of rectangles (2 across and 2 up) to partition R. (Don't worry about simplifying.)

3. (14 points) Let R be the region in the first quadrant inside $x^2 + y^2 = 4$ and below $y = x$.
[Warning: One of the following integrals below will have to be split into 2 pieces.]



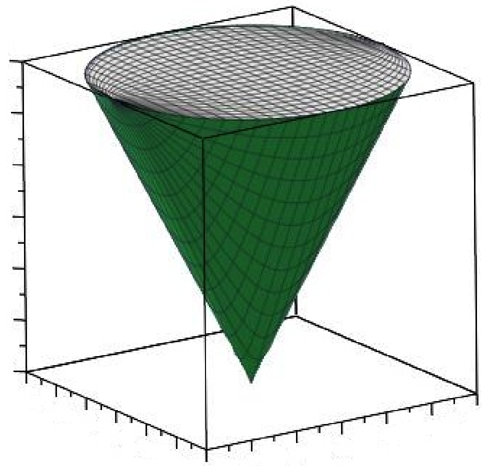
(a) Set up the integral $\iint_R x\, e^{\sqrt{x^2+y^2}}\, dA$ using the order of integration "$dy\, dx$".
[Don't evaluate the integral.]

(b) Set up the integral $\iint_R x\, e^{\sqrt{x^2+y^2}}\, dA$ using the order of integration "$dx\, dy$".
[Don't evaluate the integral.]

(c) Set up the integral $\iint_R x\, e^{\sqrt{x^2+y^2}}\, dA$ using polar coordinates.
[Don't evaluate the integral.]

4. (11 points) Compute $\iiint_E (x^2 + y^2)\, dV$ where E is bounded by $z = -1$, $z = 3$, and $x^2 + y^2 = 1$.

5. (13 points) Let E be the region bounded by $z^2 = 4x^2 + 4y^2$ and $z = 6$ A graph of this region is ever so kindly provided to the right. Set up integrals which compute the volume of E using the following order of integration and coordinate systems: [Do not evaluate these integrals.]



(a) Using the order of integation "$dz\,dy\,dx$".

(b) Using cylindrical coordinates.

(c) Using spherical coordinates.

6. (12 points) Consider the region E which is bounded by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 1$. Sketch this region then find its centroid. You should find this helpful: the volume of this region is $\dfrac{(2-\sqrt{2})\pi}{3}$. Recall that...

$$(\bar{x}, \bar{y}, \bar{z}) = \frac{1}{m}(M_{yz}, M_{xz}, M_{xy}) \qquad m = \iiint_E 1\,dV \qquad M_{yz} = \iiint_E x\,dV \qquad M_{xz} = \iiint_E y\,dV \qquad M_{xy} = \iiint_E z\,dV$$

7. (13 points) Set up the integral $\iint_R (y - x)\, dA$ where R is the region bounded by $y = x + 1$, $y = x + 3$, $y = -2x$, and $y = -2x + 4$. Use a (natural) change of coordinates which simplifies the region R and... don't forget the Jacobian!

8. (12 points) Consider the integral: $I = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{0} \int_{-\sqrt{4-x^2-y^2}}^{0} x\sqrt{x^2 + y^2 + z^2}\, dz\, dy\, dx.$

(a) Rewrite I in the following order of integration: $\iiint \quad dy\, dx\, dz$.

Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.

Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.

Do **not** evaluate the integral.