Math 2130 **Final Exam** **December 8^{th}, 2016**

Name: ______________________________ **Be sure to show your work!**

1. (14 points) Let $\mathbf{F}(x, y, z) = \langle 2xyz + 1, 2y + x^2z + 2yz^3, 3y^2z^2 + x^2y + 1 \rangle$. Also, let C be the part of the circle $x^2 + y^2 = 4$ and $z = 0$ which lies in the first quadrant and is oriented counter-clockwise.

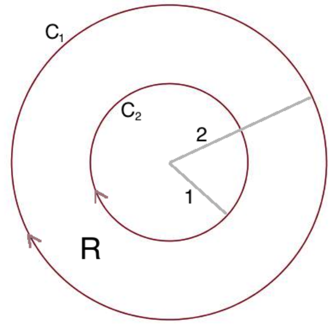
(a) Show $\mathbf{F}$ is conservative.

(b) Use the fundamental theorem of line integrals to compute $\int_C \mathbf{F} \bullet d\mathbf{r}$.

(c) Recompute $\int_C \mathbf{F} \bullet d\mathbf{r}$ directly (i.e. parameterize C etc.).

2. (8 points) C_1 is a circle of radius 2 and C_2 is a circle of radius 1 (both oriented **clockwise**). Let $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y) \rangle$ be a vector field such that P and Q have continuous first partials and in addition, $Q_x - P_y = 3$ for all points in the annulus R (between the circles C_1 and C_2). Suppose that we also know

$$\int_{C_2} P(x, y)\, dx + Q(x, y)\, dy = \pi.$$



Then $\int_{C_1} P(x, y)\, dx + Q(x, y)\, dy \quad = \quad$ ____________________.

3. (10 points) Let C be the boundary of a triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$ oriented counter-clockwise. Compute $\int_C \sin\left(\sqrt[4]{x^6+7}\right)\,dx + \left(x^2 + e^{y^4+\sin(y)}\right)\,dy$.

4. (13 points) Find the centroid of the part of the cone $z = 6 - 3\sqrt{x^2+y^2}$ which lies above the xy-plane.

Note: This is a **surface**. You should be computing **surface integrals**.

$$m = \iint_{S_1} dS \qquad M_{yz} = \iint_{S_1} x\,dS \qquad M_{xz} = \iint_{S_1} y\,dS \qquad M_{xy} = \iint_{S_1} z\,dS$$

5. (13 points) Let S_1 be parameterized by $\mathbf{r}(u,v) = \langle 3u\sin(v), u^2, 3u\cos(v)\rangle$ where $1 \le u \le 2$ and $\pi \le v \le 2\pi$.

(a) Find both orientations for S_1.

(b) Set up but **do not evaluate** the surface integral $\displaystyle\iint_{S_1} (x^3 + z)\cos(y^2)\, dS$. [Don't worry about simplifying.]

(c) Set up but **do not evaluate** the flux integral $\displaystyle\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$ where S_1 is oriented in the negative y-axis direction and $\mathbf{F}(x,y,z) = \langle x^2 + z^2, 5, x\rangle$. [Don't worry about computing the dot product or any significant simplification.]

6. (8 points) Let S_1 be the part of the sphere $x^2 + y^2 + z^2 = 25$ where $z \le 0$ and $x \ge 0$. Parameterize S_1.
Don't forget to specify bounds for your parameterization!

7. (10 points) Let S_1 be the upper hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$. Let S_2 be the disk $x^2 + y^2 \leq 4$ in the xy-plane. Orient both S_1 and S_2 upward. Suppose that $\mathbf{F}$ is a smooth vector field such that $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S} = 10$ and $\nabla \bullet \mathbf{F} = 3$.

Find $\iint_{S_2} \mathbf{F} \bullet d\mathbf{S}$.

8. (10 points) Compute the flux integral $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$ where S_1 is the unit sphere (i.e. $x^2 + y^2 + z^2 = 1$) oriented outward and $\mathbf{F}(x, y, z) = \left\langle x^3 + \sqrt{y^{10} + z^{10}}, e^{xz} + y^3, \sin(x^{15} + y + 1) + z^3 \right\rangle$.

9. (14 points) Let S_1 be the part of the paraboloid $z = x^2 + y^2$ which lies below $z = 4$. Orient S_1 downward. Verify Stokes' Theorem for the surface S_1, its boundary, and the vector field $\mathbf{F}(x, y, z) = \langle 2yz, 1, xy \rangle$.