Math 2130-101 **Test #1** **Sept. 16^{th}, 2016**

Name: ________________________________ **Be sure to show your work!**

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \qquad \mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t) \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$m = \int_C \rho\, ds \qquad (\bar{x}, \bar{y}, \bar{z}) = \frac{1}{m}\left(\int_C x\rho\, ds, \int_C y\rho\, ds, \int_C z\rho\, ds\right) \qquad \kappa = \frac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{\frac{3}{2}}}$$

1. (20 points) Vector Basics: Let $\mathbf{v} = \langle 1, 2, -3 \rangle$, $\mathbf{w} = \langle 1, -2, 2 \rangle$, and $\mathbf{u} = \langle -2, 1, 1 \rangle$.

(a) Find the area of a parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$.

(b) Compute the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

(c) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trig. functions).

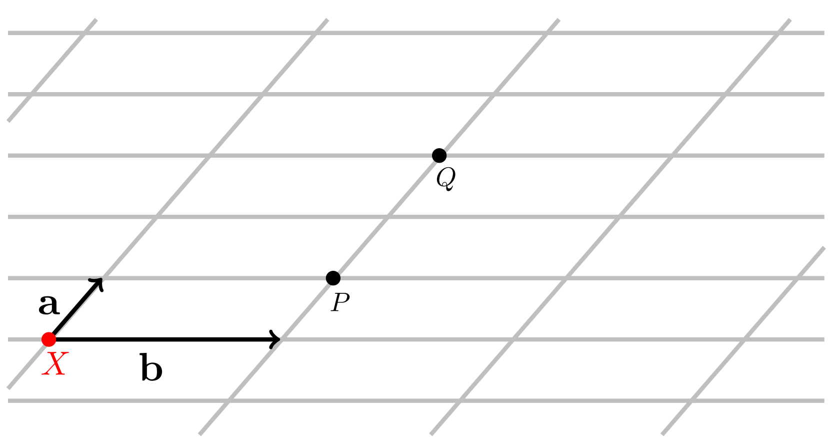
Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(d) Match with the correct response: [One of these answers doesn't occur.]

A $\mathbf{a}$ and $\mathbf{b}$ are parallel, **B** $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, **C** $\mathbf{a}$ and $\mathbf{b}$ are normalized, or **D** this is always true.

☐ $\mathbf{a} \bullet \mathbf{b} = 0$ ☐ $\mathbf{a} \bullet (\mathbf{a} \times \mathbf{b}) = 0$ ☐ $\mathbf{a} \times \mathbf{b} = \mathbf{0}$

(e) The vectors $\mathbf{a}$ and $\mathbf{b}$ are shown to the right. They are based at the point X. Sketch the vector $-2\mathbf{a} + \mathbf{b}$ based at the point P and sketch the vector $\mathbf{a} - \mathbf{b}$ based at the point Q.



2. (10 points) Let ℓ_1 be parametrized by $\mathbf{r}_1(t) = \langle 1+2t, 2-2t, 4t \rangle$ and let ℓ_2 be the line which passes through the points $P = (3,1,2)$ and $Q = (2,2,0)$. Determine if ℓ_1 and ℓ_2 are… (circle the correct answer)

the same, parallel (but not the same), intersecting, or skew.

3. (12 points) Plane old geometry.

(a) Find a (scalar) equation for the plane containing the points $A = (1,2,-1)$, $B = (3,1,1)$, and $C = (2,1,2)$.

(b) Consider the line parameterized by $\mathbf{r}(t) = \langle 1+t, 2+4t, 4-t \rangle$ and the plane $-3x + y + z = 8$. Are the line and plane parallel, perpendicular, both, or neither?

4. (9 points) Recall that the acceleration due to gravity is $\mathbf{a}(t) = -32\mathbf{k}$. Suppose that a ball is thrown starting at an initial position $\mathbf{r}_0 = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ with an initial velocity of $\mathbf{v}_0 = 3\mathbf{i} + 4\mathbf{k}$. Find the position function $\mathbf{r}(t)$ for this ball at time t .
[For what it's worth… measurements are made in feet and seconds.]

The ball's initial speed is _______ feet per second.

5. (15 points) Consider the curve C parameterized by $\mathbf{r}(t) = \langle t^3, e^{2t}, 3t \rangle$, $-1 \leq t \leq 5$.

(a) Find a parameterization, $\ell(t)$, for the line tangent to C at $t = 0$.

(b) **Set up** the line integral $\int_C x^2 y \sin(z)\, ds$. [Do **not** try to evaluate this integral! It will only end in tears.]

(c) Compute the curvature of C.

6. (9 points) Let C be the ellipse $\dfrac{(x+2)^2}{9} + \dfrac{(y-1)^2}{4} = 1$. Parameterize C and **set up** an integral which computes its arc length.

[Again, do **not** try to evaluate this integral! It will only end in tears.]

7. (14 points) Consider the curve parameterized by $\mathbf{r}(t) = \langle 3\sin(t), 4t, 3\cos(t)\rangle$.

(a) Find the TNB-frame for $\mathbf{r}(t)$.

(b) Does this curve lie in a plane? Why or why not?

(c) Find the curvature of this curve.

8. (11 points) Choose **ONE** of the following:

I. Let $\mathbf{a}$ and $\mathbf{b}$ be **unit** vectors. Show that $|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a} \bullet \mathbf{b})^2 = 1$.

[*Hint:* Use fundamental geometric identities for the dot and cross products. Don't try to do this with components.]

II. Suppose that $|\mathbf{r}(t)| = c$ (c is some constant). Show that $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are orthogonal.

Math 2130-102 **Test #1** **Sept. 16^{th}, 2016**

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$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \qquad \mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t) \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

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1. (20 points) Vector Basics: Let $\mathbf{v} = \langle 1, -3, 2 \rangle$, $\mathbf{w} = \langle -1, -2, 2 \rangle$, and $\mathbf{u} = \langle -2, 1, 3 \rangle$.

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(b) Compute the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

(c) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trig. functions).

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(d) Match with the correct response: [One of these answers doesn't occur.]

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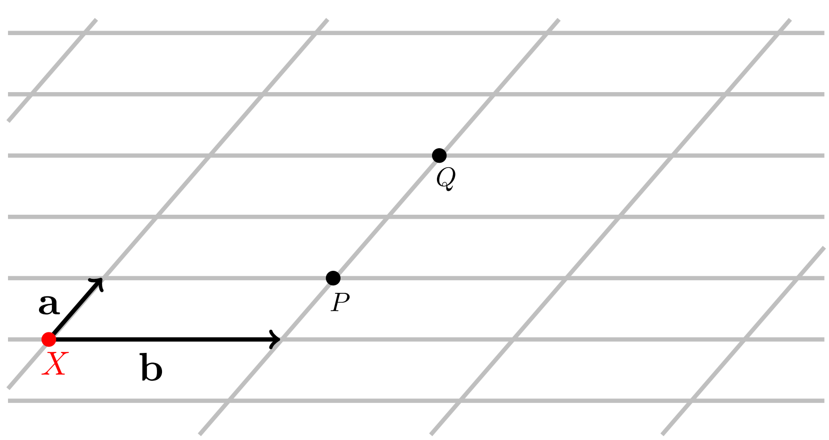
(e) The vectors $\mathbf{a}$ and $\mathbf{b}$ are shown to the right. They are based at the point X. Sketch the vector $2\mathbf{a} - \mathbf{b}$ based at the point P and sketch the vector $-\mathbf{a} + \mathbf{b}$ based at the point Q.



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4. (9 points) Recall that the acceleration due to gravity is $\mathbf{a}(t) = -32\mathbf{k}$. Suppose that a ball is thrown starting at an initial position $\mathbf{r}_0 = \mathbf{i} - 5\mathbf{k}$ with an initial velocity of $\mathbf{v}_0 = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$. Find the position function $\mathbf{r}(t)$ for this ball at time t .
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The ball's initial speed is _______ feet per second.

5. (15 points) Consider the curve C parameterized by $\mathbf{r}(t) = \langle e^{-t}, 3t, t^4 \rangle$, $-1 \leq t \leq 5$.

(a) Find a parameterization, $\ell(t)$, for the line tangent to C at $t = 0$.

(b) **Set up** the line integral $\int_C x \cos(yz^2)\, ds$. [Do **not** try to evaluate this integral! It will only end in tears.]

(c) Compute the curvature of C.

6. (9 points) Let C be the ellipse $\frac{x^2}{16} + \frac{(y-5)^2}{9} = 1$. Parameterize C and **set up** an integral which computes its arc length.

[Again, do **not** try to evaluate this integral! It will only end in tears.]

7. (14 points) Consider the curve parameterized by $\mathbf{r}(t) = \langle 3t, 4\cos(t), 4\sin(t) \rangle$.

(a) Find the TNB-frame for $\mathbf{r}(t)$.

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