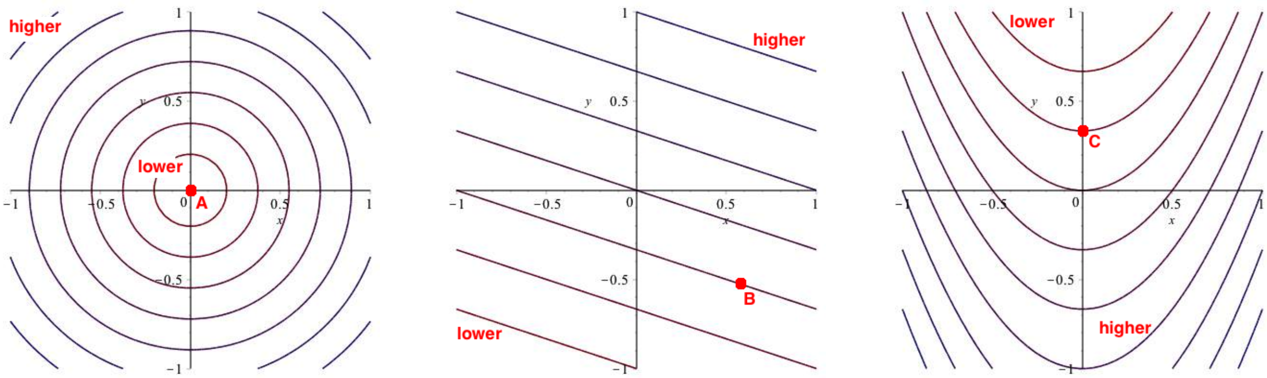
Math 2130-101 **Test #2** October 12th, 2016

Name: ______________________________ **Be sure to show your work!**

If $F(x,y)=C$, then $\dfrac{dy}{dx}=-\dfrac{F_x}{F_y}$ If $F(x,y,z)=C$, then $\dfrac{\partial z}{\partial x}=-\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y}=-\dfrac{F_y}{F_z}$

1. (12 points) Three level curve plots are shown below. I have labeled the levels so you know which curves are higher and which are lower.



(a) The plots above correspond to 3 of the functions listed here: $f(x,y)=9-x^2-y^2$, $f(x,y)=x+3y-1$, $f(x,y)=5\sqrt{x^2+y^2}$, $f(x,y)=4x^2-3y$, and $f(x,y)=4y^2-3x$. Write the correct formula below each plot.

(b) Sketch a gradient vector at the points A, B, and C. If the vector is $\mathbf{0}$ or does not exist, draw an "X" on the point. [Don't worry about having the correct length. I'm just looking for the correct direction.]

2. (6 points) Circle the correct answer and fill in the blanks.

(a) Let $f(x,y)=x+2y$. The level Curves / Surfaces of $f(x,y)$ are ______________________.

(b) Let $f(x,y)=y^2-x$. The trace of $f(x,y)$ through the yz-plane is ______________________.

3. (6 points) Let $w=f(x,y,z)$ where $x=g(t)$, $y=h(t)$, and $z=\ell(t)$. State the chain rule for the derivative of w with respect to t. Make sure you indicate which derivatives are partials and which ones are regular derivatives.

4. (6 points) Let $f(x,y,z)$ be a function with continuous **second** partials.

(a) Is $f(x,y,z)$ differentiable? **Yes / No**

(b) Could we possibly have $f_{yz}(1,2,3)=4$ and $f_{zy}(1,2,3)=5$? **Yes / No**

5. (10 points) Limits and continuity.

(a) Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{3x^2 + x^2y + 3y^2}{x^2 + y^2}$ does exist and find this limit.

(b) Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - y^2}{x^2 + y^2}$ does not exist.

6. (12 points) Let $F(x, y, z) = \sin(xz) + x^3 + y^2z$. *Note:* Both parts use the same function.

(a) Find an equation for the plane tangent to $\sin(xz) + x^3 + y^2z = 1$ at $(x, y, z) = (1, -1, 0)$

(b) Suppose z depends on x and y and that $\sin(xz) + x^3 + y^2z = 1$. Find $\dfrac{\partial z}{\partial x}$.

7. (10 points) Let $f(x, y) = x^3 + 4y$.

(a) Find the directional derivative $D_{\mathbf{u}}f(1, 2)$ where $\mathbf{u}$ points in the same direction as $\mathbf{v} = \langle -1, 1 \rangle$.

(b) What direction minimizes the directional derivative of $f(x, y)$ at the point $(x, y) = (1, 2)$? What is its minimal value?

8. (14 points) Let $f(x, y) = x^3 - xy + y + 1$.

(a) Compute the gradient and Hessian matrix for f.

(b) Find the quadratic approximation of f at $(x, y) = (2, -1)$.

(c) Find and classify all of the critical points of f. [Use the "2^{nd}-derivative" test to determine if critical points are relative max's, min's or saddle points.]

9. (12 points) Suppose $f(x, y)$ is a "nice" function (with continuous partials of all orders).

(a) $Q(x, y) = 10 + 3(x + 1)^2 + 2(x + 1)(y - 2) + 5(y - 2)^2$ is the quadratic approx. at $(x, y) = (-1, 2)$.

$\nabla f(-1, 2) = \Big\langle \qquad \Big\rangle \qquad H_f(-1, 2) = \Big[\qquad \Big]$

Is $(x, y) = (-1, 2)$ a critical point of $f(x, y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

(b) $Q(x, y) = x + 2(y - 5) + 3x^2 - x(y - 5)$ is the quadratic approx. at $(x, y) = (0, 5)$.

$\nabla f(0, 5) = \Big\langle \qquad \Big\rangle \qquad H_f(0, 5) = \Big[\qquad \Big]$

Is $(x, y) = (0, 5)$ a critical point of $f(x, y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

10. (12 points) Use the method of Lagrange multipliers to find the minimum and maximum values of $f(x, y) = 2x + 4y$ constrained to $x^2 + y^2 = 5$.

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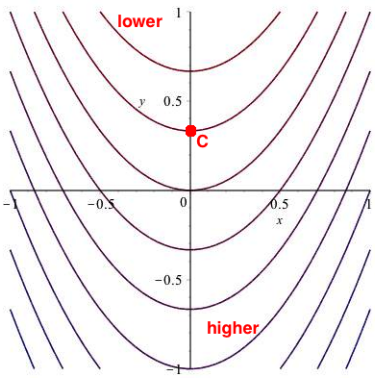
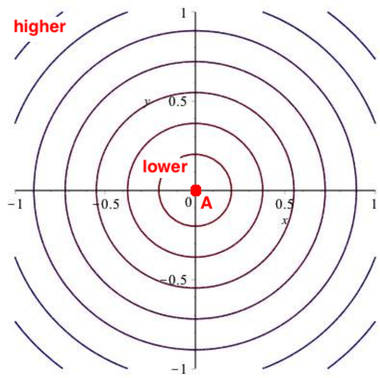
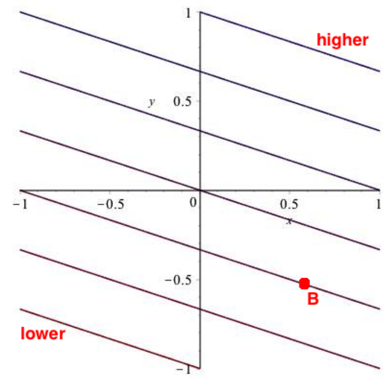
If $F(x,y)=C$, then $\dfrac{dx}{dy} = -\dfrac{F_x}{F_y}$ If $F(x,y,z)=C$, then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z}$

1. (12 points) Three level curve plots are shown below. I have labeled the levels so you know which curves are higher and which are lower.







(a) The plots above correspond to 3 of the functions listed here: $f(x,y) = 9 - x^2 - y^2$, $f(x,y) = x + 3y - 1$, $f(x,y) = 5\sqrt{x^2+y^2}$, $f(x,y) = 4x^2 - 3y$, and $f(x,y) = 4y^2 - 3x$. Write the correct formula below each plot.

(b) Sketch a gradient vector at the points A, B, and C. If the vector is **0**, draw an "X" on the point. [Don't worry about having the correct length. I'm just looking for the correct direction.]

2. (6 points) Circle the correct answer and fill in the blanks.

(a) Let $f(x,y) = x + y^2$. The level Curves / Surfaces of $f(x,y)$ are ________________________.

(b) Let $f(x,y) = y^2 - x$. The trace of $f(x,y)$ through the xz-plane is ________________________.

3. (6 points) Let $w = f(x,y,z)$ where $x = g(t)$, $y = h(t)$, and $z = \ell(t)$. State the chain rule for the derivative of w with respect to t. Make sure you indicate which derivatives are partials and which ones are regular derivatives.

4. (6 points) Let $f(x,y,z)$ be a function with continuous **second** partials.

(a) Is $f(x,y,z)$ differentiable? **Yes / No**

(b) Could we possibly have $f_{yz}(1,2,3) = 6$ and $f_{zy}(1,2,3) = 1$? **Yes / No**

5. (10 points) Limits and continuity.

(a) Show that $\lim_{(x,y)\to(0,0)} \dfrac{5x^2 + xy^2 + 5y^2}{x^2 + y^2}$ does exist and find this limit.

(b) Show that $\lim_{(x,y)\to(0,0)} \dfrac{4xy}{x^2 + y^2}$ does not exist.

6. (12 points) Let $F(x, y, z) = e^{xz} + y^2 + yz^2$. *Note:* Both parts use the same function.

(a) Find an equation for the plane tangent to $e^{xz} + y^2 + yz^2 = 7$ at $(x, y, z) = (0, 2, 1)$

(b) Suppose z depends on x and y and that $e^{xz} + y^2 + yz^2 = 7$. Find $\dfrac{\partial z}{\partial x}$.

7. (10 points) Let $f(x,y) = x^2 + 3y$.

(a) Find the directional derivative $D_{\mathbf{u}}f(2,1)$ where $\mathbf{u}$ points in the same direction as $\mathbf{v} = \langle 1, -1 \rangle$.

(b) What direction minimizes the directional derivative of $f(x,y)$ at the point $(x,y) = (2,1)$? What is its minimal value?

8. (14 points) Let $f(x,y) = x^3 - xy + y + 1$.

(a) Compute the gradient and Hessian matrix for f.

(b) Find the quadratic approximation of f at $(x,y) = (1,4)$.

(c) Find and classify all of the critical points of f. [Use the "2^{nd}-derivative" test to determine if critical points are relative max's, min's or saddle points.]

9. (12 points) Suppose $f(x, y)$ is a "nice" function (with continuous partials of all orders).

(a) $Q(x, y) = x + 2(y - 5) + 3x^2 - x(y - 5)$ is the quadratic approx. at $(x, y) = (0, 5)$.

$\nabla f(0, 5) = \Big\langle \qquad\qquad\qquad \Big\rangle$ $\qquad H_f(0, 5) = \Big[\qquad\qquad\qquad \Big]$

Is $(x, y) = (0, 5)$ a critical point of $f(x, y)$? **YES / NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

(b) $Q(x, y) = 10 + 3(x + 1)^2 + 2(x + 1)(y - 2) + 5(y - 2)^2$ is the quadratic approx. at $(x, y) = (-1, 2)$.

$\nabla f(-1, 2) = \Big\langle \qquad\qquad\qquad \Big\rangle$ $\qquad H_f(-1, 2) = \Big[\qquad\qquad\qquad \Big]$

Is $(x, y) = (-1, 2)$ a critical point of $f(x, y)$? **YES / NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

10. (12 points) Use the method of Lagrange multipliers to find the minimum and maximum values of $f(x, y) = 4x + 2y$ constrained to $x^2 + y^2 = 5$.