Math 2130-101 # Final Exam **May 2^{nd}, 2011**

Name: ______________________________________ **Be sure to show your work!**

1. (____/11 points) Lines and Planes

(a) Find an equation for the plane which contains the points: $(1, 2, 3)$, $(1, 0, 0)$, and $(-1, 2, 1)$.

(b) Find parametric equations for the line which passes through the points: $(4, 1, 6)$ and $(1, -3, 1)$.

(c) Consider the level surface $x^3 + xy + 2y^3 + yz + z^3 = 2$. Find an equation for the plane tangent to this surface at the point $(1, 0, 1)$.

2. (_____/10 points) Consider the vector valued function: $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t), t \rangle$ where $0 \leq t \leq 2\pi$.

(a) Find a formula for the curvature (i.e. $\kappa(t)$) of $\mathbf{r}(t)$.

(b) Find formulas for the unit tangent: $\mathbf{T}(t)$ and the unit normal: $\mathbf{N}(t)$ of $\mathbf{r}(t)$.

3. (_____/10 points) Big and small.

(a) Let $f(x, y) = 4xy - x^4 - y^4$. Find and classify (i.e. identify as a relative min, relative max, or saddle point) the critical points of f. *Hint:* There are 3 critical points.

(b) Set up (but do **not** solve) the equations coming from the Lagrange multipliers technique if we are trying to find the minimum and maximum value of $f(x, y, z) = xyz$ subject to the constraint $x^2 + y^2 + z^2 = 1$.

4. (_____/8 points) For each of the following vector fields, decide if $\mathbf{F}$ is consevative. Also, if it is conservative, find a potential function for $\mathbf{F}$.

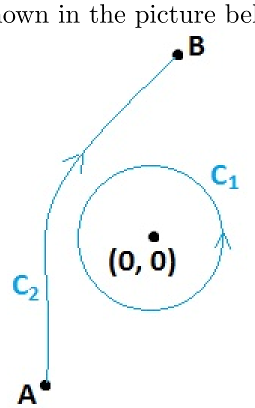
(a) $\mathbf{F}(x, y, z) = \langle x^3 + z, 3x^2 + 2yz, y^2 + x \rangle$

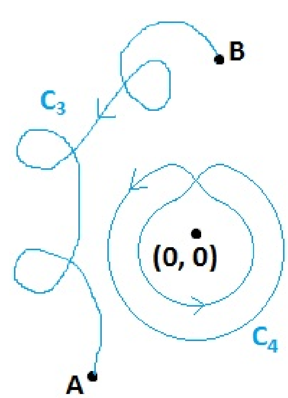
(b) $\mathbf{F}(x, y, z) = \langle y, x + 2y - z\sin(y), \cos(y) \rangle$

5. (_____/10 points) Find the centroid for the curve C: $\mathbf{r}(t) = \langle 3\cos(t) + 3,\ 3\sin(t)\rangle$, $\boxed{0 \le t \le \pi}$.

$$m = \int_C 1\,ds \qquad M_y = \int_C x\,ds \qquad M_x = \int_C y\,ds \qquad (\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

6. (_____/8 points) $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y)\rangle$ is a vector field whose component functions are continuous and have continuous partials (of all orders) everywhere except at the origin. In addition, $P_y = Q_x$ everywhere except at the origin. Suppose we know the following information: $\int_{C_1} \mathbf{F} \bullet d\mathbf{r} = 5$ and $\int_{C_2} \mathbf{F} \bullet d\mathbf{r} = 3$ where C_1 and C_2 are shown in the picture below and to the left:





Consider the curves C_3 and C_4 pictured above and to the right. Then fill in the blanks:

$\int_{C_3} \mathbf{F} \bullet d\mathbf{r} =$ ______________________ $\qquad$ $\int_{C_4} \mathbf{F} \bullet d\mathbf{r} =$ ______________________

7. (_____/10 points) Let S_1 be the part of the surface $z = 1 - x^2 - y^2$ which lies above the xy-plane (i.e. $z \geq 0$).

(a) Parameterize S_1 (remember to give bounds for your parameters).
Then find a formula for the orientation $\mathbf{n}$ of S_1 if S_1 is oriented upward.

(b) Set up, but do **not** attempt to evaluate, the surface integral $\iint_{S_1} xz\, dS$.

8. (_____/10 points) Let C be the edge of the rectangle with vertices at $(0,0)$, $(2,0)$, $(2,1)$, and $(0,1)$ oriented counter-clockwise. Evaluate $\int_C \left(\arctan(e^{-x^2}) + y^3\right)\, dx + \left(\sin(\ln(y^2+1)) + 3x\right)\, dy$
Hint: Think Green.

9. (_____/10 points) Compute the flux integral $\iint_{S_1} \mathbf{F} \bullet d\mathbf{S}$ where $\mathbf{F}(x,y,z) = xz^2\mathbf{i} + (x + z^5)\mathbf{j} + 6xy\mathbf{k}$ and S_1 is the sphere $x^2 + y^2 + z^2 = 9$ oriented outward. *Hint:* Think Divergence.

10. (_____/13 points) Let S_1 be the upper-half of the unit sphere: $x^2 + y^2 + z^2 = 1$, $z \geq 0$. Orient S_1 upward, let C be the boundary of S_1 with the induced orientation, and let $\mathbf{F}(x, y, z) = \langle y, z, x \rangle$. Verify Stoke's Theorem by computing both sides of $\int_C \mathbf{F} \bullet d\mathbf{r} = \iint_{S_1} \text{curl}(\mathbf{F}) \bullet d\mathbf{S}$