Math 2130-101 # Test #1 **February 7^{th}, 2011**

Name: ______________________________

Be sure to show your work!

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}$$

$$\kappa = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$\mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t)$$

$$\kappa = \frac{|f''(x)|}{(1+(f'(x))^2)^{\frac{3}{2}}}$$

1. (____/23 points) Let $\mathbf{u} = \langle -1, 2, 1\rangle$, $\mathbf{v} = \langle 2, -1, 2\rangle$, and $\mathbf{w} = \langle 0, 1, 2\rangle$.

(a) Compute $\text{proj}_{\mathbf{v}}(\mathbf{u})$

(b) Find the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$

(c) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trigonometric functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(d) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

(e) $\mathbf{a}$ and $\mathbf{b}$ are vectors. Match the following:

$\mathbf{a}\times\mathbf{b} = \mathbf{0}$	$\mathbf{a}$ and $\mathbf{b}$ are orthogonal.
$\mathbf{a}\cdot\mathbf{b} = 0$	This is always true.
$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{b}) = 0$	$\mathbf{a}$ and $\mathbf{b}$ are parallel.

2. (____/14 points) Lines!

(a) Find parametric equations for the line through $P = (1, 2, 3)$ and $Q = (2, -1, 1)$.

(b) Let ℓ_1 be the line parametrized by $\mathbf{r}_1(t) = \langle t + 1, 2t - 1, -t + 2\rangle$ and ℓ_2 be the line parametrized by $\mathbf{r}_2(t) = \langle 2t + 6, -t - 1, -2t - 3\rangle$. Determine if ℓ_1 and ℓ_2 are the same, parallel, intersecting, or skew.

3. (____/14 points) Planes!

(a) Find an equation for the plane which passes through the points $(1, 2, 2)$, $(3, 4, 5)$, and $(1, -1, 0)$.

(b) Find an equation for the plane which

- is parallel to the line parametrized by $\mathbf{r}(t) = \langle 2t + 1, -2t - 2, t + 3 \rangle$ and
- contains the line $\mathbf{r}(t) = \langle t + 1, t + 2, 2t - 2 \rangle$.

4. (_____/10 points) Consider the curve $\mathbf{r}(t) = \langle 5\sin(t), 4\cos(t), 3\cos(t)\rangle$ where $0 \leq t \leq 2\pi$.
Note: The original problem had ..., $2\cos(t)$, ... instead of ..., $4\cos(t)$, ... which leads to an integral which can't be done by hand.

(a) Find a formula for this curve's arc length function: $s(t)$. Also, compute the total arc length.

(b) Reparametrize this curve with respect to arc length (find "$\mathbf{r}(s)$"). Don't forget to specify the range for the arc length parameter: $?a? \leq s \leq ?b?$.

5. (_____/15 points) Find the **TNB**-frame for the helix $\mathbf{r}(t) = \langle 4\cos(t), 4\sin(t), 3t\rangle$.

6. (____/12 points) Curvature.

(a) Find a formula for the curvature of $\mathbf{r}(t) = \langle t^2, t, \sin(t) \rangle$.

(b) Suppose that $\kappa(x) = 0$ for some curve $y = f(x)$. What can you conclude about $f(x)$? What kind of curve is $y = f(x)$? Why?

7. (____/12 points) No numbers here.

(a) Choose one of the following:

I. Let $\mathbf{r}(t)$ be a vector valued function (mapping into $\mathbb{R}^3$) whose first 3 derivative exist. Compute $\frac{d}{dt}\left[\mathbf{r}\cdot(\mathbf{r}'\times\mathbf{r}'')\right]$ and simplify (get rid of any zero terms).

II. Let $\mathbf{a}$ and $\mathbf{b}$ be vectors. Show that $\mathbf{c} = \mathbf{b} - \text{proj}_{\mathbf{a}}(\mathbf{b})$ and $\mathbf{a}$ are orthogonal.

(b) $\mathbf{a}$ and $\mathbf{b}$ are pictured below. Sketch $\mathbf{a}+\mathbf{b}$, $-0.5\mathbf{b}$, and $-2\mathbf{a}+\mathbf{b}$.