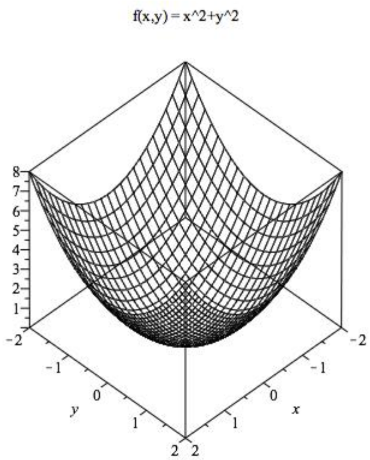
Math 2130-101 # Test #2 **March 14^{th}, 2011**

Name: ______________________________ **Be sure to show your work!**

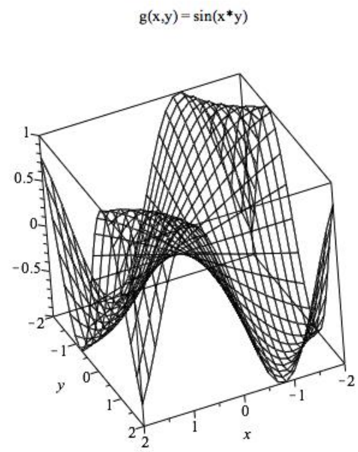
If $F(x,y) = C$, then $\dfrac{dy}{dx} = -\dfrac{F_x}{F_y}$ If $F(x,y,z) = C$, then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z}$

1. (____/12 points) Level curves and surfaces.

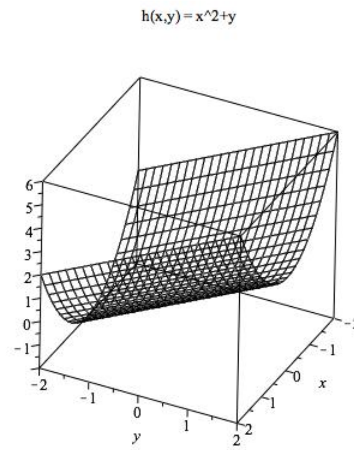
(a) Match the graph of the surface with its contour map.



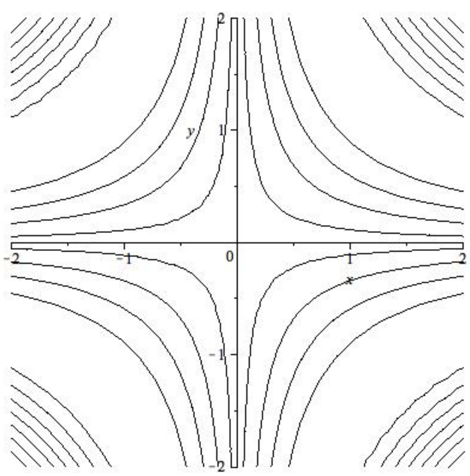
A

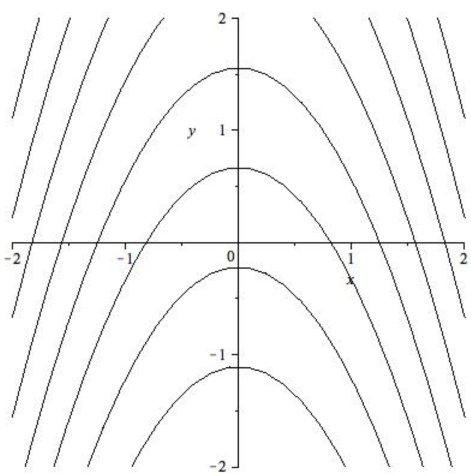


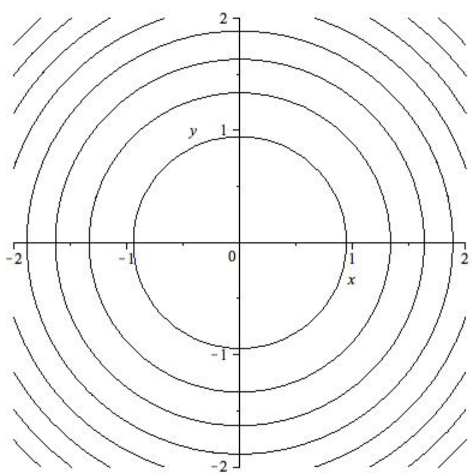
B



C







(b) State the equation for the level surfaces of $F(x,y,z) = (x-1)^2 + (y-2)^2 + (z-3)^2$.
What are these surfaces?

2. (_____/8 points) Show that the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^3 - y^3}{x^3 + y^3}$ does not exist.

3. (_____/12 points) Partials

(a) Compute $\dfrac{\partial^2 w}{\partial x \partial z}$ where $w = x^2 e^{yz}$

(b) Suppose that $x - z = \cos(yz)$. Compute the implicit derivative $\dfrac{\partial z}{\partial x}$
[*Hint:* Move everything to one side of the equation then use the given formula.]

4. (_____/12 points) Let $f(x, y) = xy^3 + 3x^2 + 1$

(a) Find the linearization of $f(x, y)$ at $(x, y) = (0, 1)$.

(b) Find the quadratic approximation of $f(x, y)$ at $(x, y) = (0, 1)$.

5. (_____/8 points) Let $w = f(x, y, z)$, $x = g(t)$, $y = h(t)$, $z = \ell(t)$. State the chain rule for $\dfrac{dw}{dt}$.

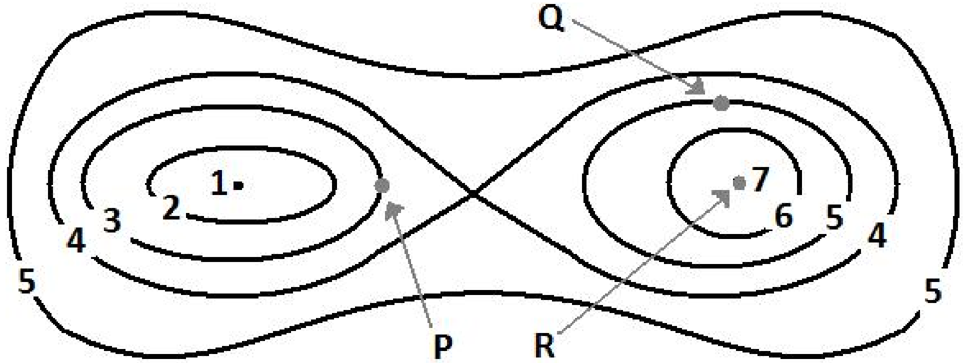
6. (_____/8 points) Find the equation of the plane tangent to $x^2z + z^2y + y^3 = -5$ at the point $(x, y, z) = (0, -1, 2)$.

7. (____/16 points) Directional Derivative.

(a) Compute $D_{\mathbf{u}} f(1, 2)$ where $f(x, y) = 2x^2 + y^3$ and $\mathbf{u}$ points in the same direction as the vector $\mathbf{v} = \langle 1, -1 \rangle$.

(b) I was working on a problem involving a very complicated function $g(x, y)$. In the course of computing my answer I found that $D_{\mathbf{u}} g(5, -2) = -12$. I also found that $\nabla g(5, -2) = \langle 3, 4 \rangle$. I know I've made a mistake. Why?

(c) Given the following contour plot. Sketch the gradient vectors at the given points or mark the point with an "X" if the gradient should be the zero vector.



8. (_____/14 points) Consider the function $f(x, y) = 4 + x^3 + y^3 - 3xy$.

(a) Find the critical points of $f(x, y)$. [*Hint:* There are only 2 critical points.]

(b) Use the "2^{nd} derivative test" to classify these points (Relative min? Relative max? Saddle point?)

9. (_____/10 points) Set up equations (coming from the Lagrange multiplier method) which allow you to find the maximum and minimum value of $f(x, y) = 4x^3 + y^2$ subject to the constraint $2x^2 + y^2 = 1$.
Do not solve the equations.