Math 2130-101 | # Test #3 | **April 8^{th}, 2011**

Name: ________________________________ **Be sure to show your work!**

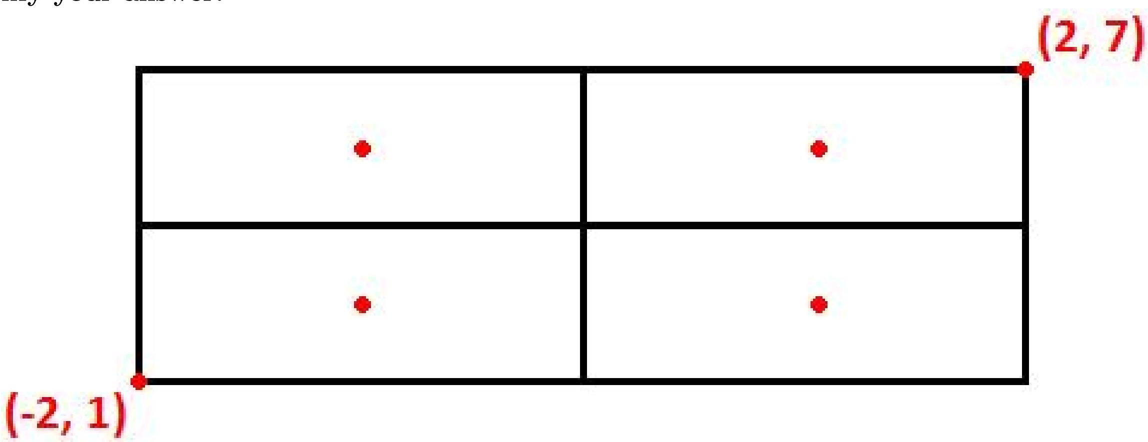
$$\begin{aligned} x &= r\cos(\theta) = \rho\cos(\theta)\sin(\phi) \\ y &= r\sin(\theta) = \rho\sin(\theta)\sin(\phi) \\ z &= z = \rho\cos(\phi) \end{aligned} \qquad J = r = \rho^2\sin(\phi)$$

$$\cos^2(\theta) = \frac{1}{2}\left(1+\cos(2\theta)\right) \qquad \sin^2(\theta) = \frac{1}{2}\left(1-\sin(2\theta)\right)$$

1. (____/12 points) Use the **midpoint** rule and a grid of 2×2 rectangles to approximate

$$\iint_R \sqrt{x^2+y}\,dA \qquad \text{where} \qquad R = [-2,2]\times[1,7]$$

First, label points on the grid shown below. Then write out the approximation. **You do not need to simplify your answer.**



2. (_____/15 points) Sketch the region of integration and evaluate $\displaystyle\int_0^2 \int_{y/2}^1 \cos(x^2)\,dx\,dy$.

Hint: It is impossible to integrate $\int \cos(x^2)\,dx$.

3. (_____/12 points) A few quick vector field questions.

(a) Let $\mathbf{F}(x,y,z) = \langle x, xy, xyz \rangle$. Compute $\text{curl}(\mathbf{F}) = \nabla \times \mathbf{F}$.

(b) Let $\mathbf{F}(x,y,z) = (x + \text{atan}(ye^z))\,\mathbf{i} + \left(x^3 + y^2 + \sqrt{z^2+5}\right)\mathbf{j} + \left(\sec(x^2+y^2) + z^3\right)\mathbf{k}$.

Compute $\text{div}(\mathbf{F}) = \nabla \bullet \mathbf{F}$.

4. (_____/17 points) Consider $\iint_R x^2\, dA$ where R is the region $\dfrac{x^2}{4} + \dfrac{y^2}{9} \leq 1$ (this is an elliptical region).

(a) Write down a helpful change of coordinates. Then compute the corresponding Jacobian matrix and its determinant. *Hint:* Modified polar should help.

$x =$ ____________________________

$y =$ ____________________________

(b) Evaluate the integral. You may find this helpful: $\displaystyle\int_0^{2\pi} \cos^2(\theta)\, d\theta = \pi$.

5. (_____/15 points) Consider the iterated integral:

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{0}\int_{0}^{\sqrt{9-x^2-y^2}} \ln(x^2+y^2+z^2+1)\,dz\,dy\,dx$$

(a) Rewrite the integral in the order of integration $\iiint \_\_\_ \, dy\, dz\, dx$.
Do not attempt to evaluate this integral.

(b) Rewrite the integral in cylindrical coordinates.
Do not attempt to evaluate this integral.

(c) Rewrite the integral in spherical coordinates.
Do not attempt to evaluate this integral.

6. (_____/15 points) Compute the centroid of the solid cone bounded by $z = \sqrt{x^2 + y^2}$ and $z = 2$.
Hint: Use symmetry to reduce the number of integrals needed. The volume of this solid is $(8/3)\pi$.

$$(\bar{x}, \bar{y}, \bar{z}) = \underline{\hspace{6cm}}$$

7. (_____/14 points) Evaluate the integral $\iiint_E \sqrt{x^2 + y^2 + z^2}\, dV$ where E is the solid which lies above the xy-plane (i.e. $z \geq 0$) and is bounded by the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 1$.