Math 2130-101

Test #1

February 15^{th}**, 2012**

Name: ________________________________

Be sure to show your work!

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}$$

$$\kappa = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$\mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t)$$

$$\kappa = \frac{|f''(x)|}{(1 + (f'(x))^2)^{\frac{3}{2}}}$$

1. (____/15 points) Let $\mathbf{v} = \langle 2, -1, 1\rangle$, and $\mathbf{w} = \langle -3, 1, 2\rangle$.

(a) Compute the projection of $\mathbf{v}$ along $\mathbf{w}$: $\text{proj}_{\mathbf{w}}(\mathbf{v})$.

(b) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trigonometric functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(c) Find the area of the triangle $\triangle PQR$ whose vertices are the points $P = (1, 0, 1)$, $Q = (2, 1, 2)$, and $R = (1, 2, 3)$.

2. (____/14 points) Line & Plane

(a) Let ℓ_1 be the line parametrized by $\mathbf{r}_1(t) = \langle -1+t, -t, -1-2t \rangle$ and ℓ_2 be the line parametrized by $\mathbf{r}_2(t) = \langle 3+t, 2+2t, 1+3t \rangle$. Determine if ℓ_1 and ℓ_2 are the same, parallel, intersecting, or skew.

(b) Find an equation for the plane which is passes through the points: $P = (1, 2, 0)$, $Q = (2, 1, 1)$, and $R = (-1, 3, 2)$.

3. (_____/14 points) Parametrizations and such.

(a) Consider the curve parametrized by $\mathbf{r}(t) = \langle 4\cos(t) - 3, 2\sin(t)\rangle$. Find a parametrization $\ell(t)$ for the line tangent to this curve at $t = \pi/3$. [*Note:* $\cos(\pi/3) = 1/2$ and $\sin(\pi/3) = \sqrt{3}/2$.]

What is kind of curve is this? (Be specific) ____

(b) Find a parametrization $\mathbf{r}(t)$ for the line segment from $P = (-1, 1, 2)$ to $Q = (1, 2, 1)$. Don't forget to specify bounds for the parameter t: $??? \leq t \leq ???$.

4. (____/15 points) Consider the curve $\mathbf{r}(t) = \langle 3\sin(t), 3\cos(t), 4t \rangle$ where $0 \le t \le 4\pi$.

(a) Find the arc length of this curve. [*Hint:* The integral you end up with should be easy to evaluate.]

(b) Find the **TNB**-frame.

5. (____/14 points) Consider the twisted cubic: $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$.

(a) Find the curvature of $\mathbf{r}(t)$.

(b) Find the tangential and normal components of acceleration.

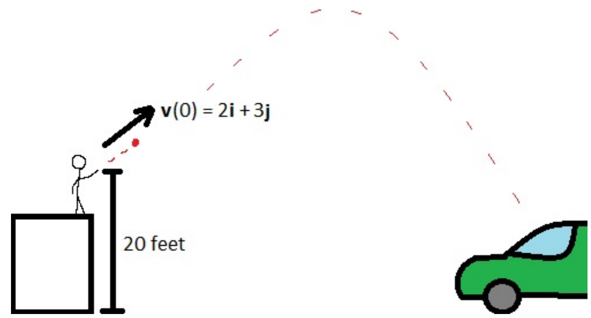
$a_T =$ ____________________ $a_N =$ ____________________

6. (____/14 points) Bob threw a ball off the top of a 20 foot tall building (so $\mathbf{r}(0) = 20\mathbf{j}$). The ball's initial velocity vector was $\mathbf{v}(0) = 2\mathbf{i} + 3\mathbf{j}$. Recall that the acceleration due to gravity is $\mathbf{a}(t) = -32\mathbf{j}$ (ft/s^2).

What was the ball's initial speed? ______________________________ feet per second.

Find the formula for $\mathbf{r}(t)$.

Irrelevant Picture:



Irrelevant picture criticism: The velocity vector is not accurately rendered. Also, what does that car have to do with the problem? Hey! The car almost 20 feet tall!?! and Bob is a giant! That must be his car. Wait, why is he trying to break his own windshield? Must be insurance fraud. *Conclusion:* Physics is bad.

7. (____/14 points) No numbers here.

(a) Choose **ONE** of the following:

I. Suppose $\mathbf{v}$ and $\mathbf{w}$ have the same length. Show $\mathbf{v}+\mathbf{w}$ and $\mathbf{v}-\mathbf{w}$ are perpendicular.

II. Suppose that $y = f(x)$ has zero curvature. Show that $y = f(x)$ is a line.

(b) $\mathbf{a}$ and $\mathbf{b}$ are pictured below. Sketch $-2\mathbf{a}$ and $\mathbf{a}-\mathbf{b}$.