Math 2130-101 **Test #2** **March 20^{th}, 2012**

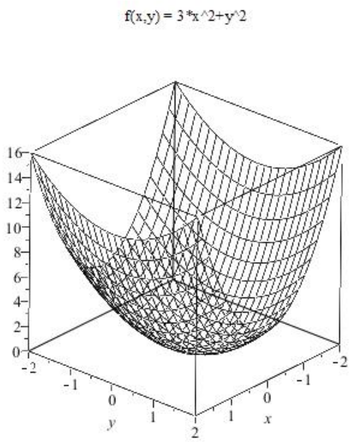
Name: ______________________________________ **Be sure to show your work!**

If $F(x,y) = C$, then $\dfrac{dy}{dx} = -\dfrac{F_x}{F_y}$

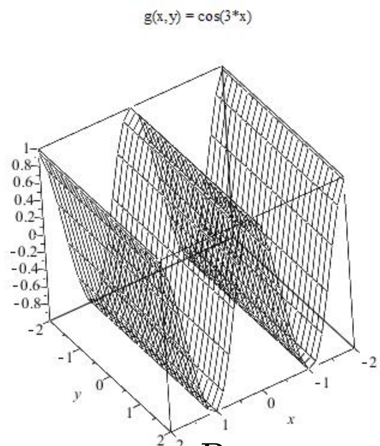
If $F(x,y,z) = C$, then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z}$

1. (____/9 points) Level curves and surfaces.

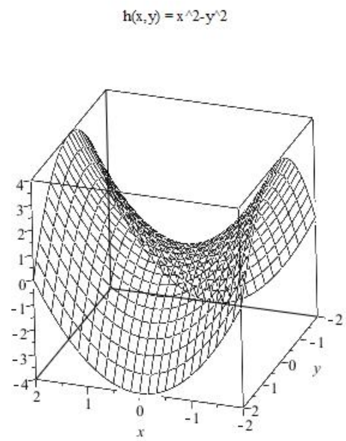
(a) Match the graph of the surface with its contour map.



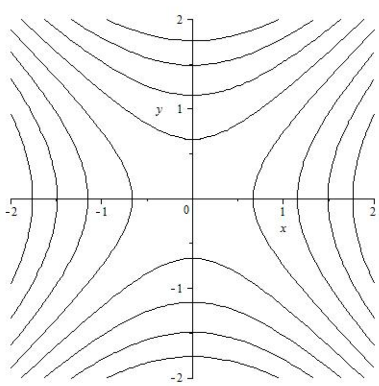
A

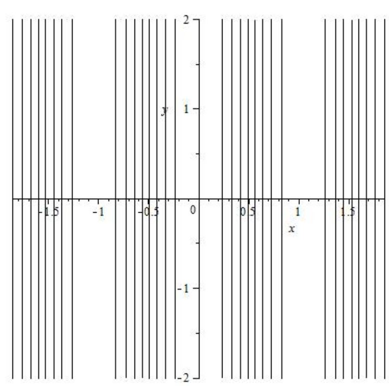


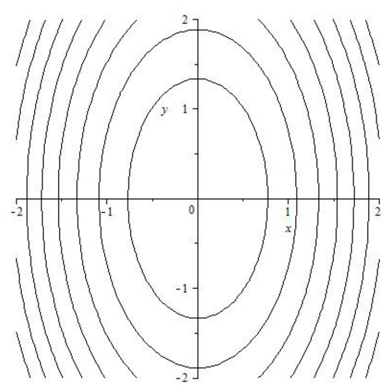
B



C







(b) State the equation for the level surfaces of $F(x,y,z) = x^2 + y^2 + z^2$.

Such surfaces are called __________________________________

(c) State the equation for the level surfaces of $F(x,y,z) = 3x - 2y + 5z$.

Such surfaces are called __________________________________

2. (_____/9 points) Show that the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x^2+3xy+y^2}$ does not exist.

3. (_____/10 points) The volume enclosed by a pipe of radius R and length L is given by $V = \pi R^2 L$. Use a differential to approximate the maximal error in measured volume if the measurement of the radius R is off by at most 2% and measurement of the length L is off by at most 3%.

4. (_____/9 points) Let $w = f(x, y, z)$, $x = g(u, v)$, $y = h(u, v)$, $z = \ell(u, v)$. State the chain rule for $\dfrac{\partial w}{\partial u}$.

5. (_____/12 points) Find the quadratic approximation for $f(x, y) = x^3 + 4xy - 2y$ based at $(x, y) = (1, -1)$.

6. (_____/10 points) Find the equation of the plane tangent to $xy^2 + e^{yz} - z = 3$ at $(x, y, z) = (1, 0, -2)$.

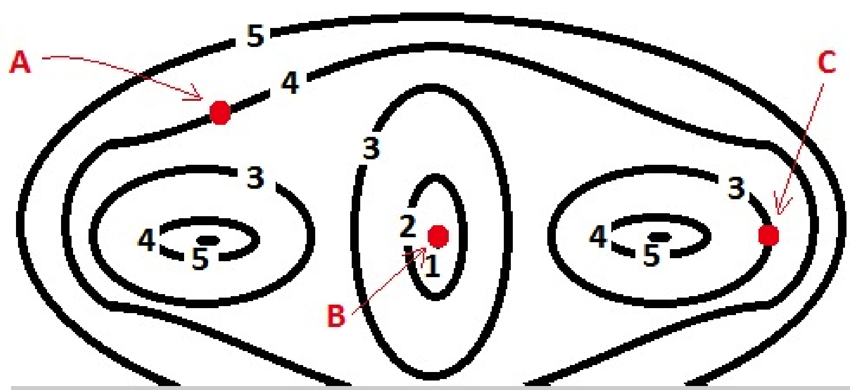
7. (_____/10 points) Approximate $\iint\limits_R f(x, y)\, dA$ (the volume under the surface) with a Riemann sum if $f(x, y) = xy^2$, $R = [-2, 2] \times [0, 8]$. Use a total of 4 rectangular boxes (a 2×2 grid) and the midpoint rule.

8. (_____/12 points) Directional Derivative.

(a) Compute $D_{\mathbf{u}}f(1,0)$ where $f(x,y) = x^3 + 2xy + 1$ and $\mathbf{u}$ points in the same direction as $\mathbf{v} = \langle -3, 4 \rangle$.

(b) Find the minimum **value** of the directional derivative of f at $(x,y) = (1,0)$.

(c) Given the following contour plot. Sketch the gradient vectors at the given points or mark the point with an "X" if the gradient should be the zero vector.



The gradient at C is not clearly determined. Skip it.

Circle the letters corresponding to critical points. Then for each critical point write "local min, local max, saddle point, or nothing" below its letter (according to which type of point it is).

A B C

9. (_____/10 points) Suppose that $f(x, y)$ is a function which has continuous partials of all orders (everywhere).

(a) How should you check to see if $(-3, 1)$ is a critical point of $f(x, y)$?

(b) Suppose that $(2, 5)$ is a critical point of $f(x, y)$. Given $f_{xx}(2, 5) = -2$, $f_{xy}(2, 5) = 1$, and $f_{yy}(2, 5) = -3$, write down the Hessian of $f(x, y)$ at $(x, y) = (2, 5)$. What kind of critical point is this? [Relative min, relative max, saddle point, or not enough information.]

(c) Same as part (b), given $f_{xx}(2, 5) = 1$, $f_{xy}(2, 5) = 2$, and $f_{yy}(2, 5) = 4$.

10. (_____/9 points) Set up equations (coming from the Lagrange multiplier method) which allow you to find the maximum and minimum value of $f(x, y, z) = xy^2z + 2yz$ subject to the constraint $x^2 + 2y^2 + 3z^2 = 1$. **Do not solve the equations.**