Math 2130 # Test #3 **April 18^{th}, 2012**

Name: ______________________________ **Be sure to show your work!**

$$\begin{aligned} x &= \rho\cos(\theta)\sin(\varphi) \\ y &= \rho\sin(\theta)\sin(\varphi) \\ z &= \rho\cos(\varphi) \end{aligned}$$

$$J = \rho^2\sin(\varphi)$$

$$\cos^2(\theta) = \frac{1}{2}\left(1+\cos(2\theta)\right)$$

1. (____/14 points) Consider $\int_0^4 \int_{\sqrt{x}}^2 \sin(y^3)\,dy\,dx$.
Sketch the region of integration and then **evaluate** the integral. *Hint:* $\int \sin(y^3)\,dy$ is impossible to evaluate.

2. (____/14 points) Consider the integral $\iint\limits_R (2x+y)\cos(3x-y)\,dA$ where R is bounded by $y=-2x$, $y=-2x+1$, $y=3x-2$, and $y=3x-3$. State a change of coordinates: $u =???$ and $v =???$ so that the resulting integral can be evaluated. Perform the change of coordinates, but do **not** evaluate the integral.

3. (_____/15 points) Consider the integral: $I = \int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{0} \int_{-\sqrt{16-x^2-y^2}}^{0} 5x \, dz \, dy \, dx.$

(a) Rewrite I in the following order of integration: $\iiint \quad dx \, dz \, dy$.

Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.

Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.

Do **not** evaluate the integral.

4. (_____/14 points) Find the centroid of the region E where E is bounded below by $z = x^2 + y^2$ and above by $z = 8 - x^2 - y^2$. *Hint:* Use symmetry to cut down the number of integrals you need to evaluate. You should find the following fact useful: The volume of E is 16π.

$$m = \iiint_E 1 \, dV \qquad M_{yz} = \iiint_E x \, dV \qquad M_{xz} = \iiint_E y \, dV \qquad M_{xy} = \iiint_E z \, dV$$

5. (_____/14 points) Evaluate $\iiint\limits_E x\,dV$ where E is bounded by $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.

6. (_____/14 points) Evaluate $\iiint\limits_E \sqrt{x^2 + y^2 + z^2}\,dV$ where E is the region in the first octant (i.e. $x, y, z \geq 0$) which is outside $x^2 + y^2 + z^2 = 1$ and inside $x^2 + y^2 + z^2 = 4$.

7. (_____/15 points) A few vector fields.

(a) One of the following vectors fields is conservative and the other is not. Circle the conservative vector field and then find a potential function, $f(x, y)$.

$$\mathbf{F}(x, y) = \langle x^2 + y^2, x^2 + y \rangle \qquad\qquad \mathbf{F}(x, y) = \langle 2xy + 6x + 1, x^2 + \cos(y) \rangle$$

(b) Compute $\nabla \bullet \mathbf{F}$ and $\nabla \times \mathbf{F}$ where $\mathbf{F}(x, y, z) = \langle 2xyz, x^2z + z + 2, x^2y + y \rangle$. Is $\mathbf{F}$ conservative? ____________

(c) Compute $\nabla \bullet \mathbf{F}$ and $\nabla \times \mathbf{F}$ where $\mathbf{F}(x, y, z) = \langle x^2 + y, y^2 + x, e^{yz} \rangle$. Is $\mathbf{F}$ conservative? ____________

(d) Of the vectors fields from (b) and (c), one is conservative. Find a potential function for it.