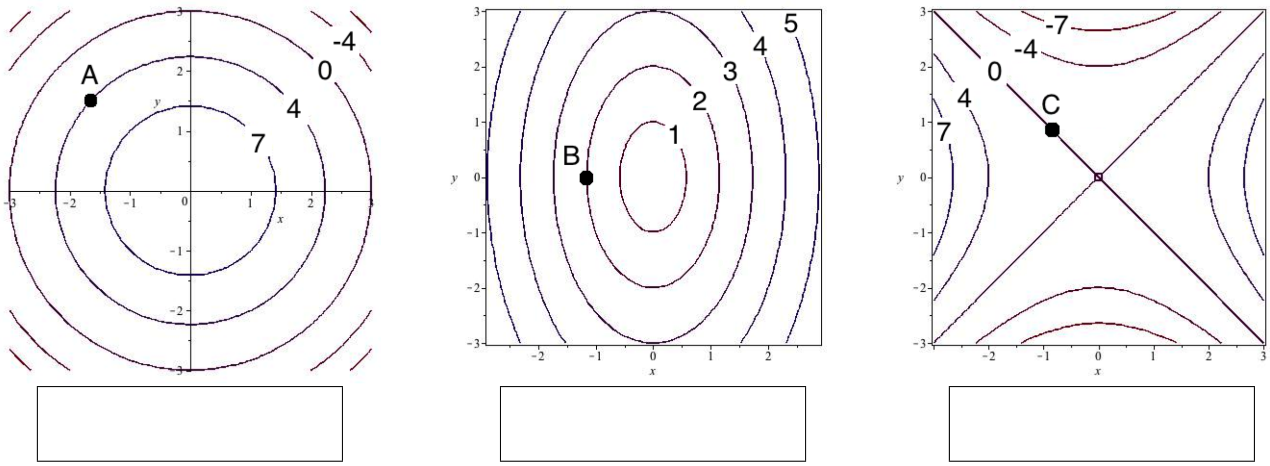
Math 2130-101 # Test #2 **March 19^{th}, 2013**

Name: ______________________________ **Be sure to show your work!**

If $F(x,y)=C$, then $\dfrac{dy}{dx}=-\dfrac{F_x}{F_y}$ If $F(x,y,z)=C$, then $\dfrac{\partial z}{\partial x}=-\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y}=-\dfrac{F_y}{F_z}$

1. (12 points) Three level curve plots are shown below. I have labeled the levels so you know which curves are higher and which are lower.



(a) The plots above correspond to the functions: $f(x,y)=x^2-y^2$, $f(x,y)=9-x^2-y^2$, and $f(x,y)=\sqrt{3x^2+y^2}$. Write the correct formula below each plot.

(b) Sketch a gradient vector at the points A, B, and C. If the vector is $\mathbf{0}$, draw an "X" on the point. [Don't worry about having the correct length. I'm just looking for the correct direction.]

2. (8 points) State the **chain rule** for the derivative or partial derivative (whichever makes sense) of w with respect to t where $w=f(x,y,z)$, $x=g(t)$, $y=h(t)$, and $z=\ell(t)$. Make sure you clearly label regular derivatives with d's and partials with ∂'s. If your handwriting leaves this difficult to determine, write "regular" and "partial" and draw arrows to which is which.

3. (10 points) Show the following limit does not exist: $$\lim_{(x,y)\to(0,0)} \frac{x^2+3y^2}{x^2+xy}$$

4. (10 points) Suppose that $z = \dfrac{x^3}{y^2}$. We know that x is off by no more than 2% and y by no more than 1%. Use a differential to approximate the percent error in z.

5. (10 points) Find an equation for the plane tangent to $xe^{xyz} + y^2 z = -1$ at the point $(x, y, z) = (0, 1, -1)$.

6. (8 points) Suppose we have a function of two variables: $f(x, y)$. For each question circle **YES** or **NO**. Then briefly explain your answer (in a sentence or two).

(a) Is it possible to have $f_{xy} \neq f_{yx}$? **YES** / **NO**

(b) Suppose f_x and f_y exist everywhere. Can I conclude that f is differentiable? **YES** / **NO**

7. (12 points) Let $f(x,y) = x^2 + xy + 3y + 1$.

(a) Find the gradient of f and the Hessian matrix of f.

(b) Find the quadratic approximation of f at $(x,y) = (-1,0)$.

(c) Find an classify the critical point(s) of $f(x,y)$.
[Use the "2^{nd}-derivative" test to determine if critical points are relative max's, min's or saddle points.]

8. (10 points) Let $f(x,y) = x^2y^3 + 2x + 1$

(a) Find the directional derivative of f at the point $(x,y) = (1,1)$ and in the same direction as $\mathbf{v} = \langle -2, 3 \rangle$.

(b) Can the directional derivative of f at the point $(x,y) = (1,1)$ be equal to 5? **YES / NO**

Can it be equal to -10? **YES / NO**

Briefly explain your answer.

9. (10 points) Suppose $f(x,y)$ is a "nice" function (with continuous partials of all orders).

(a) $Q(x,y) = 2(x-1) + 3(y-2) + (x-1)^2 + 3(x-1)(y-2) + 3(y-2)^2$ is the quadratic approx. at $(x,y) = (1,2)$.

$\nabla f(1,2) = \Big\langle \qquad\qquad\qquad\qquad \Big\rangle \qquad H_f(1,2) = \Bigg[\qquad\qquad\qquad\qquad \Bigg]$

Is $(x,y) = (1,2)$ a critical point of $f(x,y)$? **YES / NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

(b) $Q(x,y) = 5 + (-3)(x-3)^2 + 2(x-3)(y+2) + (-3)(y+2)^2$ is the quadratic approx. at $(x,y) = (3,-2)$.

$\nabla f(3,-2) = \Big\langle \qquad\qquad\qquad\qquad \Big\rangle \qquad H_f(3,-2) = \Bigg[\qquad\qquad\qquad\qquad \Bigg]$

Is $(x,y) = (3,-2)$ a critical point of $f(x,y)$? **YES / NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

10. (10 points) Set up but **do not solve** the equations used in the method of Lagrange multipliers for finding the minimum and maximum values of $f(x,y,z) = xyz$ constrained to $x^2 + 2y^2 + 3z^2 = 4$.