Math 2130-101

Test #3

April 9^{th}, 2013

Name: ________________________ **Be sure to show your work!**

$$\begin{aligned} x &= \rho\cos(\theta)\sin(\varphi) \\ y &= \rho\sin(\theta)\sin(\varphi) \\ z &= \rho\cos(\varphi) \end{aligned}$$

$$J = \rho^2 \sin(\varphi)$$

$$\cos^2(\theta) = \frac{1}{2}\left(1+\cos(2\theta)\right)$$

1. (14 points) Use a double Riemann sum to approximate $\displaystyle\iint_R y^2 e^{-x}\,dA$ where $R = [-4,2]\times[-1,5]$.

 Use midpoint rule and a 3×2 grid of rectangles (3 across and 2 up) to partition R. (Don't worry about simplifying.)

2. (14 points) First, sketch the region of integration and then evaluate $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1}\,dx\,dy$.

 Hint: $\int \sqrt{x^3+1}\,dx$ cannot be expressed in terms of elementary functions – that is – you can't integrate it.

3. (14 points) Find the centroid of $R = \{(x, y) \mid x^2 + y^2 \leq 9 \text{ and } x \geq 0\}$ (the right-half of the disk of radius 3 centered at the origin). Feel free to use what you know about areas of circles and symmetry to cut down the number of integrals you need to evaluate.

$$m = \iint_R 1 \, dA \qquad M_y = M_{x=0} = \iint_R x \, dA \qquad M_x = M_{y=0} = \iint_R y \, dA$$

4. (15 points) Consider the integral: $I = \int_{-5}^{5} \int_{0}^{\sqrt{25-x^2}} \int_{-\sqrt{25-x^2-y^2}}^{0} y + 3 \, dz \, dy \, dx.$

(a) Rewrite I in the following order of integration: $\iiint \quad dx \, dz \, dy$.

Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.

Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.

Do **not** evaluate the integral.

5. (14 points) Consider the double integral $\iint_R \frac{-2x+y}{x+3y}\, dA$ where R is bounded by the lines $y = 2x+1$, $y = 2x+5$, $y = -\frac{1}{3}x + 1$, and $y = -\frac{1}{3}x + 2$. Use the change of variables $u = -2x + y$ and $v = x + 3y$ to rewrite the double integral as an iterated intergral (with order of integration $du\, dv$). Don't forget the Jacobian!!! **Do not evaluate this integral!**

6. (14 points) Consider the region E bounded above by $z = 4 - x^2 - y^2$, bounded below by the xy-plane ($z \geq 0$), and in front of the xz-plane ($y \geq 0$).

(a) Write $\iiint_E x^2\, dV$ as an iterated integral with order of integration $\iiint \quad dy\, dz\, dx$.

(b) Write $\iiint_E x^2\, dV$ in terms of cylindrical coordinates and evaluate the integral.

7. (15 points) Let E be the region bounded below by $z = \sqrt{x^2 + y^2}$ and above by $z = 3$.

(a) Rewrite the equations: $z = \sqrt{x^2 + y^2}$ and $z = 3$ in terms of cylindrical coordinates.

(b) Rewrite the equations: $z = \sqrt{x^2 + y^2}$ and $z = 3$ in terms of spherical coordinates.

(c) Write $\iiint_E ze^{-x^2-y^2-z^2}\, dV$ in terms of spherical coordinates. **Do not evaluate this integral!**