Name:

Be sure to show your work!

$$\begin{array}{rcl} x & = & \rho\cos(\theta)\sin(\varphi) & J = \rho^2\sin(\varphi) \\ y & = & \rho\sin(\theta)\sin(\varphi) \\ z & = & \rho\cos(\varphi) & \cos^2(\theta) = \frac{1}{2}\left(1 + \cos(2\theta)\right) \end{array}$$

1. (14 points) Use a double Riemann sum to approximate $\iint_R y^2 e^{-x} dA$ where $R = [-4, 2] \times [-1, 5]$.

Use midpoint rule and a 3×2 grid of rectangles (3 across and 2 up) to partition R. (Don't worry about simplifying.)

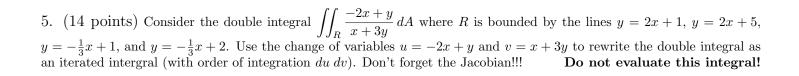
2. (14 points) First, sketch the region of integration and then evaluate $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3 + 1} \, dx \, dy$. Hint: $\int \sqrt{x^3 + 1} \, dx$ cannot be expressed in terms of elementary functions – that is – you can't integrate it. 3. (14 points) Find the centroid of $R = \{(x,y) | x^2 + y^2 \le 9 \text{ and } x \ge 0\}$ (the right-half of the disk of radius 3 centered at the origin). Feel free to use what you know about areas of circles and symmetry to cut down the number of integrals you need to evaluate.

$$m = \iint_R 1 \, dA$$
 $M_y = M_{x=0} = \iint_R x \, dA$ $M_x = M_{y=0} = \iint_R y \, dA$

- 4. (15 points) Consider the integral: $I = \int_{-5}^{5} \int_{0}^{\sqrt{25-x^2}} \int_{-\sqrt{25-x^2-y^2}}^{0} y + 3 dz dy dx$.
- (a) Rewrite I in the following order of integration: $\iiint dx dz dy$. Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates. Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates. Do **not** evaluate the integral.



6. (14 points) Consider the region E bounded above by
$$z = 4 - x^2 - y^2$$
, bounded below by the xy-plane $(z \ge 0)$, and in front of the xz-plane $(y \ge 0)$.

- (a) Write $\iiint_E x^2 dV$ as an iterated integral with order of integration $\iiint_E dy dz dx$.
- (b) Write $\iiint_E x^2 dV$ in terms of cylindrical coordinates and evaluate the integral.

- 7. (15 points) Let E be the region bounded below by $z = \sqrt{x^2 + y^2}$ and above by z = 3.
- (a) Rewrite the equations: $z = \sqrt{x^2 + y^2}$ and z = 3 in terms of cylindrical coordinates.

(b) Rewrite the equations: $z = \sqrt{x^2 + y^2}$ and z = 3 in terms of spherical coordinates.

(c) Write $\iiint_E z e^{-x^2-y^2-z^2}\,dV$ in terms of spherical coordinates.

Do not evaluate this integral!