Math 2130 **Test #1** **February 12^{th}, 2016**

Name: ______________________________ **Be sure to show your work!**

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \qquad \mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t) \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$m = \int_C \rho\, ds \qquad (\bar{x}, \bar{y}, \bar{z}) = \frac{1}{m}\left(\int_C x\rho\, ds, \int_C y\rho\, ds, \int_C z\rho\, ds\right) \qquad \kappa = \frac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{\frac{3}{2}}}$$

1. (20 points) Vector Basics: Let $\mathbf{u} = \langle 2, -2, 1\rangle$, $\mathbf{v} = \langle -1, 3, 1\rangle$, and $\mathbf{w} = \langle -1, -1, 0\rangle$.

(a) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

(b) Find a vector that points in the same direction as $\mathbf{u}$ but has length 5.

(c) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trig. functions).

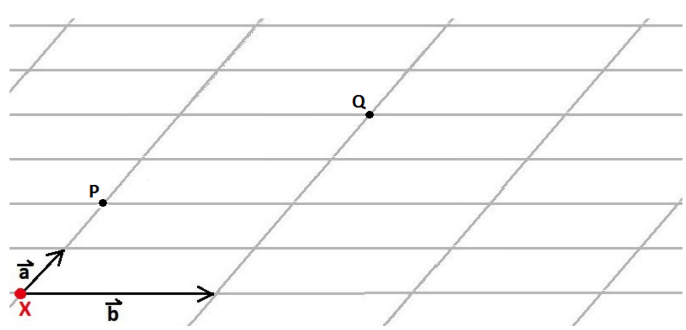
Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(d) Fill in the blanks ($\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors)...

(i) "$\mathbf{a} \times \mathbf{b} = \mathbf{0}$" tells us that $\mathbf{a}$ and $\mathbf{b}$ are ______________________________.

(ii) "$\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c}) = 0$" tells us that $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are ______________________________.

(e) The vectors $\mathbf{a}$ and $\mathbf{b}$ are shown to the right. They are based at the point X. Sketch the vector $\mathbf{a} + \mathbf{b}$ based at the point P and sketch the vector $\mathbf{b} - \mathbf{a}$ based at the point Q.



2. (10 points) Let ℓ_1 be parametrized by $\mathbf{r}_1(t) = \langle t, -t+1, 3t+2\rangle$ and let ℓ_2 be the line which passes through the points $P = (-1, 2, -1)$ and $Q = (2, 1, 0)$. Determine if ℓ_1 and ℓ_2 are... (circle the correct answer)

the same, parallel (but not the same), intersecting, or skew.

3. (12 points) Plane old geometry.

(a) Find a (scalar) equation for the plane that passes through the points $A = (2, 1, -1)$, $B = (3, 2, 1)$, and $C = (2, 3, 2)$.

(b) Find the area of the triangle with vertices A, B, and C (as in part (a)).

4. (10 points) A particle moves with constant acceleration $\mathbf{a}(t) = 2\mathbf{i} + 4\mathbf{k}$ (meters per second^2). Initially its velocity is $\mathbf{v}_0 = 3\mathbf{i} - 4\mathbf{j}$ (meters per second) and it begins at position $\mathbf{r}_0 = \mathbf{i} + \mathbf{j}$ (meters). Find the position function $\mathbf{r}(t)$ for this particle (t is measured in seconds).

What is the particle's initial speed? __________________________ (meters per second).

5. (10 points) Let C be the circle $(x+3)^2 + (y-2)^2 = 4$. Parameterize C and then compute its arc length.
[You must compute an integral – don't just use geometry.]

6. (12 points) Let C be parameterized by $\mathbf{r}(t) = \langle \sin(t), t, e^t \rangle$ where $-2 \leq t \leq 7$.

(a) Find the curvature of $\mathbf{r}(t)$.

(b) Set up the line integral $\int_C (x^2 e^y + \cos(z))\, ds$. [Do not try to evaluate this integral. It will only end in tears.]

7. (14 points) Consider the curve parameterized by $\mathbf{r}(t) = \langle 4t, 3\cos(t), 3\sin(t)\rangle$.

(a) Parameterize a line tangent to $\mathbf{r}(t)$ at $t = \pi$.

(b) Find the TNB-frame for $\mathbf{r}(t)$.

Does this curve lie in a plane? Why or why not?

8. (12 points) Choose **ONE** of the following: [In both cases, drawing a good explanatory picture will earn you some partial credit – but for full credit you need more.]

I. Suppose $\mathbf{v}$ and $\mathbf{w}$ have the same length. Show $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$ are perpendicular.

II. Let C be a point and ℓ a line parameterized by $\mathbf{r}(t) = A + \overrightarrow{AB}\, t$. Explain why the distance from the point C to the line ℓ is given by $\dfrac{|\overrightarrow{AC} \times \overrightarrow{AB}|}{|\overrightarrow{AB}|}$.