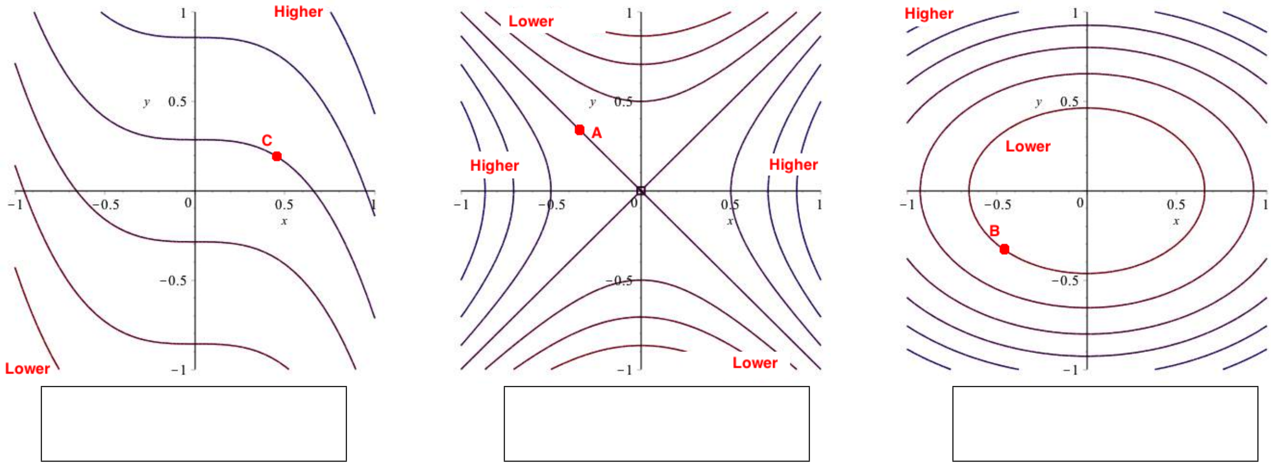
Name: ______________________________ **Be sure to show your work!**

If $F(x,y)=C$, then $\dfrac{dy}{dx}=-\dfrac{F_x}{F_y}$ If $F(x,y,z)=C$, then $\dfrac{\partial z}{\partial x}=-\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y}=-\dfrac{F_y}{F_z}$

1. (12 points) Three level curve plots are shown below. I have labeled the levels so you know which curves are higher and which are lower.



(a) The plots above correspond to 3 of the functions listed here: $f(x,y)=10-x^2-2y^2$, $f(x,y)=x^2+2y^2$, $f(x,y)=x^2-y^2$, $f(x,y)=y+x^3$, and $f(x,y)=y+x^2$. Write the correct formula below each plot.

(b) Sketch a gradient vector at the points A, B, and C. If the vector is **0**, draw an "X" on the point. [Don't worry about having the correct length. I'm just looking for the correct direction.]

2. (6 points) Circle the correct answer and fill in the blanks.

(a) Let $f(x,y,z)=3x-y+5z$. The level Curves / Surfaces of $f(x,y,z)$ are ______________________.

(b) Let $f(x,y)=9-x^2-y^2$. The trace of $f(x,y)$ through the xz-plane is ______________________.

3. (8 points) Let $w=f(x,y,z)$ where $x=g(t)$, $y=h(t)$, and $z=\ell(t)$. State the chain rule for the derivative of w with respect to t. Make sure you indicate which derivatives are partials and which ones are regular derivatives.

4. (12 points) Limits and continuity.

(a) Show the function $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 2 & (x,y) = (0,0) \end{cases}$ is **not** continuous at the origin.

(b) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$ does exist and find this limit.

5. (15 points) Let $F(x,y,z) = xy^2 + ze^{xy}$. *Note:* All three parts use the same function and point.

(a) Find an equation for the plane tangent to $xy^2 + ze^{xy} = 3$ at $(x,y,z) = (0,2,3)$

(b) Find the directional derivative $D_{\mathbf{u}}F(0,2,3)$ where $\mathbf{u}$ points in the same direction as $\mathbf{v} = \langle 2,-2,1\rangle$.

(c) Could $D_{\mathbf{u}}F(0,2,3) = -10$ for some direction $\mathbf{u}$? **Yes** / **No** Why or why not?

6. (8 points) Create a diagram showing how the following statements about $f(x, y)$ are related:

(1) f has continuous first partials (2) the first partials of f exist (3) f is continuous (4) f is differentiable

For example: "(1) $\Longleftarrow$ (2) $\Longleftrightarrow$ (3) $\Longrightarrow$ (4)" is a wrong answer.

7. (15 points) Let $f(x, y) = x^2y + 2xy$.

(a) Compute the gradient and Hessian matrix for f.

(b) Find the quadratic approximation of f at $(x, y) = (1, 2)$.

(c) Find and classify all of the critical points of f. [Use the "2^{nd}-derivative" test to determine if critical points are relative max's, min's or saddle points.]

8. (12 points) Suppose $f(x,y)$ is a "nice" function (with continuous partials of all orders).

(a) $Q(x,y) = -4+2(x-3)+7(y-1)+2(x-3)^2-(x-3)(y-1)+3(y-1)^2$ is the quadratic approx. at $(x,y)=(3,1)$.

$\nabla f(3,1) = \Big\langle \qquad \Big\rangle$ $\qquad H_f(3,1) = \Big[\qquad \Big]$

Is $(x,y)=(3,1)$ a critical point of $f(x,y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

(b) $Q(x,y) = 2-2(x+1)^2+(x+1)y-5y^2$ is the quadratic approx. at $(x,y)=(-1,0)$.

$\nabla f(-1,0) = \Big\langle \qquad \Big\rangle$ $\qquad H_f(-1,0) = \Big[\qquad \Big]$

Is $(x,y)=(-1,0)$ a critical point of $f(x,y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

9. (12 points) Use the method of Lagrange multipliers to find the minimum and maximum values of $f(x,y)=x^2+4y$ constrained to $x^2+y^2=9$.