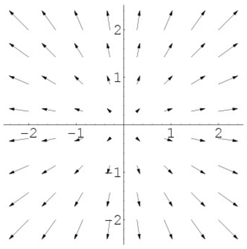
Math 2130 # Test #3 **April 18^{th}, 2016**

Name: ________________________________ **Be sure to show your work!**

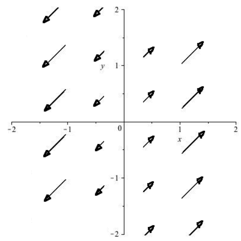
$$x = \rho\cos(\theta)\sin(\varphi) \qquad J = \rho^2\sin(\varphi)$$
$$y = \rho\sin(\theta)\sin(\varphi)$$
$$z = \rho\cos(\varphi) \qquad \sin^2(\theta) = \tfrac{1}{2}\left(1-\cos(2\theta)\right)$$

1. (13 points) A few vector fields.

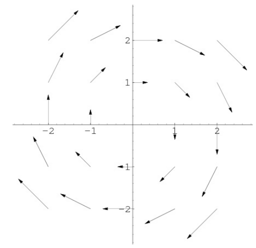
(a) The following are plots of several vector fields. Please note that all of the vectors have been scaled down so they do not overlap each other. Write A, B, and C next to the appropriate vector field's formula. Put an X next to the formula whose vector field is **not shown**. Also, for each vector field is **F** conservative? Circle "Yes" or "No".



A



B



C

☐ $\mathbf{F}(x,y) = \left\langle \frac{x}{5}, \frac{x}{5} \right\rangle$	☐ $\mathbf{F}(x,y) = \langle y, -x\rangle$	☐ $\mathbf{F}(x,y) = \langle -y, -x\rangle$	☐ $\mathbf{F}(x,y) = \langle x, y\rangle$
Yes / No	Yes / No	Yes / No	Yes / No

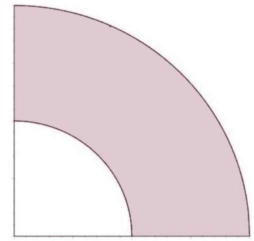
(b) Compute the divergence and curl of $\mathbf{F}(x,y,z) = \langle x^2y + z, y - xy^2, xy - z\rangle$. [Show your work!]

Is **F** conservative? Yes / No

2. (8 points) Use a double Riemann sum to approximate $\iint\limits_R e^x \sin(y)\, dA$ where $R = [-6,6]\times[0,8]$.

Use midpoint rule and a 2×2 grid of rectangles (2 across and 2 up) to partition R. (Don't worry about simplifying.)

3. (14 points) Let R be the region inside $x^2 + y^2 = 4$, outside $x^2 + y^2 = 1$, and in the first quadrant. [**Warning:** One of the following integrals below will have to be **split into 2 pieces**.]



(a) Set up the integral $\displaystyle\iint_R y\sqrt{x^2+y^2}\,dA$ using the order of integration "$dy\,dx$".

[Don't evaluate the integral.]

(b) Set up the integral $\displaystyle\iint_R y\sqrt{x^2+y^2}\,dA$ using polar coordinates.

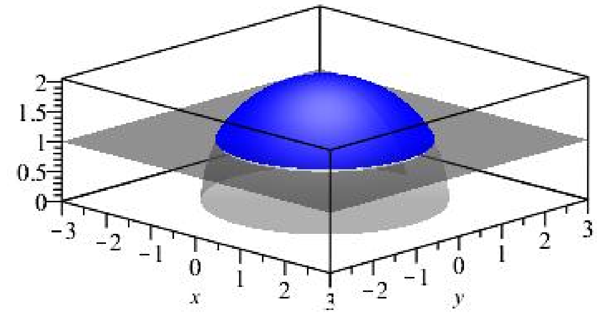
[Don't evaluate the integral.]

(c) Evaluate the integral $\displaystyle\iint_R y\sqrt{x^2+y^2}\,dA$.

4. (13 points) Let E be the region inside $x^2 + y^2 + z^2 = 1$ and under the xy-plane (i.e. $z \le 0$). So E is the lower half of the unit ball. Find the centroid of E. *Hint:* Use symmetry and geometry to cut down the number of necessary integrals. Also, the volume inside a sphere of radius R is $\frac{4}{3}\pi R^3$.

$$(\bar{x}, \bar{y}, \bar{z}) = \frac{1}{m}(M_{yz}, M_{xz}, M_{xy}) \qquad m = \iiint_E 1\,dV \qquad M_{yz} = \iiint_E x\,dV \qquad M_{xz} = \iiint_E y\,dV \qquad M_{xy} = \iiint_E z\,dV$$

5. (14 points) Let E be the region inside $x^2+y^2+z^2=4$ and above $z=1$. A graph of this region is ever so kindly provided to the right. Set up integrals which compute the volume of E using the following order of integration and coordinate systems: [**Do not evaluate** these integrals.]



(a) Using the order of integation "$dz\,dy\,dx$".

(b) Using cylindrical coordinates.

(c) Using spherical coordinates.

6. (13 points) Let E be the region above $z=x^2+y^2$ and below $z=2-x^2-y^2$ and **where y** $\geq$ **0**. Evaluate $\displaystyle\iiint_E y\,dV$.

7. (13 points) Set up the integral $\iint_R (x-y)\sin(x+y)\,dA$ where R is the region bounded by $y=-x$, $y=-x+2$, $y=x-1$, and $y=x-3$. Use a (natural) change of coordinates which simplifies the region R and simplifies the function being integrated. Also, don't forget the Jacobian! [**Do not** try to evaluate this integral.]

8. (12 points) Consider the integral: $I = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{0} \int_{0}^{\sqrt{4-x^2-y^2}} z\, e^{x^2+y^2+z^2}\, dz\, dy\, dx.$

(a) Rewrite I in the following order of integration: $\iiint \quad dy\, dx\, dz$.

Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.

Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.

Do **not** evaluate the integral.