Math 2130-101 # Test #1 **February 14^{th}, 2018**

Name: ______________________________ **Be sure to show your work!**

$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \dfrac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}$ $\mathbf{r}''(t) = \left(\dfrac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t)$ $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$

$m = \int_C \rho\, ds$ $(\bar{x}, \bar{y}, \bar{z}) = \dfrac{1}{m}\left(\int_C x\rho\, ds, \int_C y\rho\, ds, \int_C z\rho\, ds\right)$ $\kappa = \dfrac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{\frac{3}{2}}}$

1. (24 points) Vector Basics: Let $\mathbf{u} = \langle -1, -1, 1\rangle$, $\mathbf{v} = \langle 1, 0, 3\rangle$, and $\mathbf{w} = \langle 2, -1, 2\rangle$.

(a) Compute the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$.

(b) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

(c) Find two unit vectors which are parallel to $\mathbf{w}$.

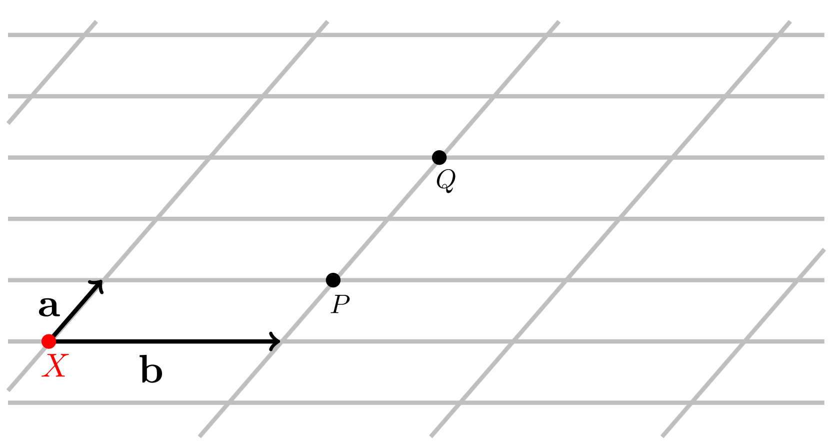
(d) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trig. functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(e) Match the statement on the left to the corresponding statement on the right...

☐	$(\mathbf{a} \times \mathbf{b}) \bullet \mathbf{b} = 0$	**A)** cannot ever be true / is nonsense
☐	$\mathbf{a} \bullet \mathbf{b} = \mathbf{a} \times \mathbf{b}$	**B)** is always true
☐	$\mathbf{a} \times \mathbf{b} = \mathbf{0}$	**C)** $\mathbf{a}$ and $\mathbf{b}$ are orthogonal
☐	$\mathbf{a} \bullet \mathbf{b} = 0$	**D)** $\mathbf{a}$ and $\mathbf{b}$ are parallel

(f) The vectors $\mathbf{a}$ and $\mathbf{b}$ are shown to the right. They are based at the point X. Sketch the vector $\mathbf{a} + \mathbf{b}$ based at the point P and sketch the vector $2\mathbf{a} - \mathbf{b}$ based at the point Q.



2. (10 points) Let ℓ_1 be parametrized by $\mathbf{r}_1(t) = \langle 1, 1, 3\rangle + \langle 2, 1, -1\rangle t$ and let ℓ_2 be the line which passes through the points $P = (2, 0, 2)$ and $Q = (3, 4, 2)$. Determine if ℓ_1 and ℓ_2 are... (circle the correct answer)

the same, parallel (but not the same), intersecting, or skew.

3. (14 points) Plane old geometry.

(a) Find the (scalar) equation of the plane through the points $A = (1, 2, 0)$, $B = (3, 3, 1)$, and $C = (2, 1, 1)$.

(b) The planes: $x + 2y + 3z = 4$ and $5x - y - z = 1$ are... [Circle **ALL** that apply.]

parallel perpendicular intersecting the same

~~5~~ **4. (14 points)** Parameterization, arc length, and a line integral.

Let C be the **upper-half** of the circle $x^2 + y^2 = 4$. Parameterize C, compute its arc length (with an integral – not just using geometry), and then set up $\int_C y\, e^{x^2+y^2}\, ds$.

~~6~~ **5. (13 points)** Let C be parameterized by $\mathbf{r}(t) = \langle t^2, -t^2, t^3 \rangle$.

(a) Find the curvature of $\mathbf{r}(t)$.

(b) Find the tangential and normal components of acceleration for $\mathbf{r}(t)$ (i.e. $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$).

~~7~~ **6. (13 points)** Find the TNB-frame for the curve C parameterized by $\mathbf{r}(t) = \langle 3\sin(t), 5\cos(t), 4\sin(t)\rangle$.

Find the curvature of C

Does this curve lie in a plane? Why or why not?

~~8~~ **7. (12 points)** No numbers here. Choose **ONE** of the following:

I. Suppose that $\mathbf{v}$ and $\mathbf{w}$ have the same length. Show that $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$ are orthogonal. Draw a picture to go with your proof.

II. Given vectors $\mathbf{a}$ and $\mathbf{b}$, is it always true that $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}|$? Either explain why this always holds or explain what would make it hold or fail.