Math 2130 | **Test #3** | **April 16$^{\text{th}}$, 2018**

Test #3

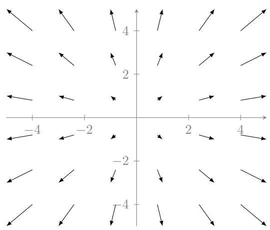
Name: ______________________________ **Be sure to show your work!**

$$\begin{aligned} x &= \rho\cos(\theta)\sin(\varphi) \\ y &= \rho\sin(\theta)\sin(\varphi) \\ z &= \rho\cos(\varphi) \end{aligned}$$

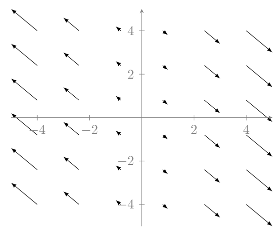
$J = \rho^2\sin(\varphi)$

$\cos^2(\theta) = \frac{1}{2}\left(1+\cos(2\theta)\right)$

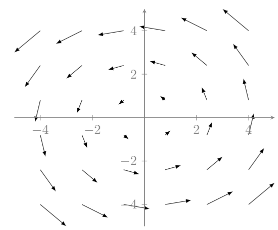
1. (12 points) The following are plots of several vector fields. Please note that all of the vectors have been scaled down so they do not overlap each other. Write A, B, and C next to the appropriate vector field's formula. Put an X next to the formula whose vector field is **not shown**. Also, for each vector field is **F** conservative? Circle "Yes" or "No".



A



B



C

☐ $\mathbf{F}(x,y) = \langle x, -x\rangle$ Yes / No

☐ $\mathbf{F}(x,y) = \langle y, x\rangle$ Yes / No

☐ $\mathbf{F}(x,y) = \langle -y, x\rangle$ Yes / No

☐ $\mathbf{F}(x,y) = \langle x, y\rangle$ Yes / No

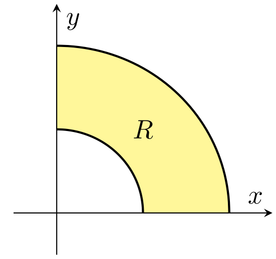
2. (9 points) Let $\mathbf{F}(x,y,z) = \left\langle x+y^2z^3,\ y+x^5z,\ \sin(y)+z\right\rangle$. Compute the divergence and curl of $\mathbf{F}$. Determine if the vector field is conservative.

This vector field **IS / IS NOT** conservative.

3. (9 points) Use a double Riemann sum to approximate $\displaystyle\iint_R \cos(x+y^2)\,dA$ where $R = [-2,2]\times[0,6]$.

Use midpoint rule and a 2×2 grid of rectangles (2 across and 2 up) to partition R. (Don't worry about simplifying.)

4. (10 points) Let R be the region in the first quadrant bounded outside $x^2 + y^2 = 1$ and inside $x^2 + y^2 = 4$. [Warning: One of the integrals below will have to be split into 2 pieces.]



(a) Set up the integral $\iint_R 2\cos(x^2 + y^2)\, dA$ using the order of integration "$dy\, dx$".

(b) Set up the integral $\iint_R 2\cos(x^2 + y^2)\, dA$ using polar coordinates.

Do not evaluate this integral.

5. (12 points) Evaluate $\displaystyle\int_0^6 \int_{y/2}^3 e^{x^2}\, dx\, dy$. Include a sketch of the associated region of integration.

Note/Hint: You cannot integrate $\int e^{x^2}\, dx$ in terms of elementary functions.

6. (12 points) Set up $\iint_R (x-y)^2 e^{x-y}\, dA$ where R is bounded by $x=0$, $2x+y=1$, $x-y=0$, and $x-y=2$. Use the change of coordinates: $u = x-y$ and $v = 2x+y$ and...don't forget the Jacobian!

Do not evaluate this integral.

7. (12 points) Consider the region E above $z = 2x^2+2y^2$ and below $z = 3-x^2-y^2$. Compute the centroid of E.

Note: The volume of E is $\frac{3}{2}\pi$. Also, you may use symmetry help cut down your work.

$$m = \iiint_E 1\, dV \qquad M_{yz} = \iiint_E x\, dV \qquad M_{xz} = \iiint_E y\, dV \qquad M_{xy} = \iiint_E z\, dV \qquad (\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m}\right)$$

8. (10 points) Consider the integral: $I = \int_{-5}^{0} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \int_{0}^{\sqrt{25-x^2-y^2}} \frac{y}{\sqrt{x^2+y^2+z^2}} \, dz \, dy \, dx.$

(a) Rewrite I in the following order of integration: $\iiint \quad dy \, dx \, dz$.

Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.

Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.

Do **not** evaluate the integral.

9. (14 points) Let E be the region inside $x^2 + y^2 + z^2 = 4$ and above $z = 1$. Set up integrals which compute the volume of E using the following orders of integration: [Do **not** evaluate these integrals.]

(a) $\int_?^? \int_?^? \int_?^? ??? \, dz \, dy \, dx$

(b) Set up this integral in cylindrical coordinates.

(c) Set up this integral in spherical coordinates.