Math 2130-101 # Final Exam **May 3rd, 2021**

Name: ______________________________ **Be sure to show your work!**

1. (6 points) Let $\mathbf{F}(x, y, z) = \langle \ln(x^3 + 1), y\sin(xz), x^2y + z^5 \rangle$. Compute $\nabla \times \mathbf{F}$ and $\nabla \bullet \mathbf{F}$.

2. (11 points) Let $\mathbf{F}(x, y, z) = \big\langle 2x + z, 1 + 2yz, y^2 + x \big\rangle$, and let C be the line segment from $(0, 1, 1)$ to $(2, 1, 0)$.

Note: $\mathbf{F}$ is a conservative vector field (I've checked for you).

(a) Use the **fundamental theorem of line integrals** to compute $\int_C \mathbf{F} \bullet d\mathbf{r}$.

(b) Recompute $\int_C \mathbf{F} \bullet d\mathbf{r}$ directly (i.e. parameterize C etc.).

3. (5 points) Suppose $\mathbf{F} = \langle M, N, P\rangle$ is a vector field where M, N, P have continuous partial derivatives of all orders defined on all of $\mathbb{R}^3$. Fill in the blank and circle the correct answers.

If $\nabla \times \mathbf{F} = \mathbf{0}$, then $\mathbf{F}$ is a ____________________________ vector field.

$\nabla \times \mathbf{F} = \mathbf{0}$ implies $\mathbf{F}$'s line / flux integrals are path / surface independent.

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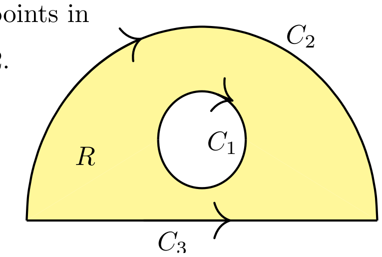
4. (9 points) Let C be the counter-clockwise oriented boundary of the region bounded by $y = 1$ and $y = 2 - x^2$.

Compute $\displaystyle\int_C \left(-y^2 + \sqrt{x^6 + e^{3x}}\right) dx + \sin(y^8 + ye^y)\, dy$.

5. (9 points) C_1 is a circle of radius 1 (oriented clockwise), C_2 is an upper-half of a circle of radius 3 (oriented clockwise), and C_3 is a line segment closing off the semi-circle (oriented left to right). Let $\mathbf{F}(x,y) = \langle M(x,y), N(x,y)\rangle$ be a vector field such that M and N have continuous first partials and in addition, $N_x - M_y = 4$ for all points in region R. We also know $\displaystyle\int_{C_2} \mathbf{F} \bullet d\mathbf{r} = \pi$ and $\displaystyle\int_{C_3} \mathbf{F} \bullet d\mathbf{r} = 5\pi$. *Note:* The area of R is $7\pi/2$.



Compute $\displaystyle\int_{C_1} M(x,y)\, dx + N(x,y)\, dy \quad = \quad$ ________________________.

6. (10 points) Let S_1 be the upper hemisphere: $x^2 + y^2 + z^2 = 4$, $z \geq 0$ and S_2 be the disk $x^2 + y^2 \leq 4$ in the xy-plane. Orient both S_1 and S_2 upward. Let $\mathbf{F}$ be a smooth vector field such that $\iint_{S_2} \mathbf{F} \bullet \mathbf{n}\, d\sigma = \pi$ and $\nabla \bullet \mathbf{F} = z$.

Find $\iint_{S_1} \mathbf{F} \bullet \mathbf{n}\, d\sigma$.

7. (12 points) Let S_1 be the surface parameterized by $\mathbf{r}(u, v) = \langle v \sin(u), v \cos(u), 2v \rangle$ where $-\pi/2 \leq u \leq 2\pi$ and $4 \leq v \leq 11$.

(a) Find both orientations for S_1.

(b) Set up but **do not evaluate** the surface integral $\displaystyle\iint_{S_1} \sqrt{x^2 + y^2} \cdot e^z \, d\sigma$. [Don't worry about simplifying.]

(c) Set up but **do not evaluate** the flux integral $\displaystyle\iint_{S_1} \mathbf{F} \bullet \mathbf{n}\, d\sigma$ where S_1 is oriented downward and $\mathbf{F}(x, y, z) = \langle z, y^2, 9 + x \rangle$.
[Don't worry about computing the dot product or any significant simplification.]

8. (12 points) Find the centroid of the part of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant (i.e., $x, y, z \geq 0$).

Note: This is a **surface**. You should be computing **surface integrals**.

$$m = \iint_{S_1} 1\, d\sigma \qquad M_{yz} = \iint_{S_1} x\, d\sigma \qquad M_{xz} = \iint_{S_1} y\, d\sigma \qquad M_{xy} = \iint_{S_1} z\, d\sigma$$

9. (11 points) Consider the solid cylinder E: $x^2 + y^2 \leq 1$ and $1 \leq z \leq 3$ and let $S_1 = \partial E$ be its outward oriented surface. In addition, let $\mathbf{F}(x, y, z) = \left\langle xz + \sqrt[6]{y^4 + z^4 + 12},\ yz + \sin^{10}(x + z^2),\ \ln(x^8 + y^2 + 99) \right\rangle$. Compute the flux integral $\iint_{S_1} \mathbf{F} \bullet \mathbf{n}\, d\sigma$.

10. (15 points) Let S_1 be the surface $z = x^2 + y^2$, $1 \le z \le 4$ (i.e., part of a circular paraboloid). Orient S_1 downward. Verify Stokes' Theorem for the surface S_1, its boundary, and the vector field $\mathbf{F} = \langle y + x^2, y, xz + 5 \rangle$.