Math 2130-101 **Test #1** **Feb.** 19^{th}**, 2021**

Name: ______________________________ **Be sure to show your work!**

$\text{proj}_{\mathbf{w}}(\mathbf{v}) = \dfrac{\mathbf{v} \bullet \mathbf{w}}{|\mathbf{w}|^2}\mathbf{w}$ $\quad$ $\mathbf{r}''(t) = \left(\dfrac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t)$ $\quad$ $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$

$m = \displaystyle\int_C \delta\, ds$ $\quad$ $(\bar{x}, \bar{y}, \bar{z}) = \dfrac{1}{m}\left(\displaystyle\int_C x\delta\, ds, \int_C y\delta\, ds, \int_C z\delta\, ds\right)$ $\quad$ $\tau = \dfrac{(\mathbf{r}'(t) \times \mathbf{r}''(t)) \bullet \mathbf{r}'''(t)}{|\mathbf{r}'(t) \times \mathbf{r}''(t)|^2}$ $\quad$ $\kappa = \dfrac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{\frac{3}{2}}}$

1. (22 points) Vector Basics: Let $\mathbf{u} = \langle 2, -2, 1\rangle$, $\mathbf{v} = \langle -1, 3, 1\rangle$, and $\mathbf{w} = \langle 1, 1, 0\rangle$.

(a) Find two unit vectors that are perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.

(b) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

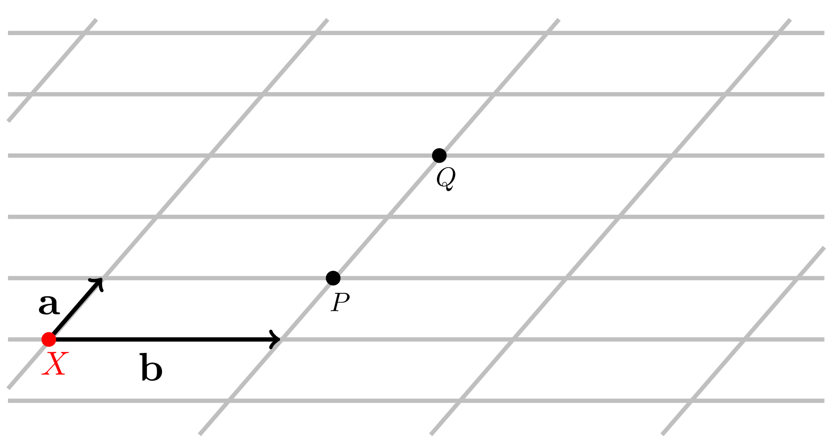
(c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ (don't worry about evaluating inverse trig. functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(d) Match the statement on the left to the corresponding statement on the right...

☐	$\mathbf{a} \bullet \mathbf{b} = 0$	**A)** $\mathbf{a}$ and $\mathbf{b}$ are parallel
☐	$\mathbf{a} \times \mathbf{b} = \mathbf{0}$	**B)** $\mathbf{a}$ and $\mathbf{b}$ are orthogonal
☐	$(\mathbf{a} \bullet \mathbf{b}) \times (\mathbf{a} \bullet \mathbf{b}) = \mathbf{0}$	**C)** is always true
☐	$(\mathbf{a} \times \mathbf{b}) \bullet \mathbf{b} = 0$	**D)** is nonsense

(e) The vectors $\mathbf{a}$ and $\mathbf{b}$ are shown to the right. They are based at the point X. Sketch the vector $-\mathbf{a} + \mathbf{b}$ based at the point P and sketch the vector $2\mathbf{a} - \mathbf{b}$ based at the point Q.



2. (8 points) Let ℓ_1 be parametrized by $\mathbf{r}_1(t) = \langle t, -t+1, 3t+2 \rangle$ and let ℓ_2 be the line which passes through the points $P = (-1, 2, -1)$ and $Q = (2, 1, 0)$. Determine if ℓ_1 and ℓ_2 are... (circle the correct answer)

the same, parallel (but not the same), intersecting, or skew.

3. (12 points) Plane old geometry.

(a) Find a (scalar) equation for the plane containing the points $A = (2, 1, -1)$, $B = (3, 2, 1)$, and $C = (2, 3, 2)$.

(b) Find the area of the triangle ΔABC where A, B, and C are the same points as in part (a).

4. (10 points) A strange object is observed to have velocity function $\mathbf{v}(t) = 3t^2\mathbf{i} - 6\mathbf{j} + e^t\mathbf{k}$. In addition, this object's initial position was known to be $\mathbf{r}_0 = 5\mathbf{i} + 100\mathbf{k}$. [For what it's worth... measurements are made in meters and seconds.]

(a) This object's acceleration function is $\mathbf{a}(t) =$ ______________________________

(b) Find this object's position function $\mathbf{r}(t)$.

Its initial speed was _______ meters per second.

5. (10 points) Parameterize and set up an integral that computes the arc length of the ellipse $\dfrac{(x-1)^2}{4} + \dfrac{(y+2)^2}{9} = 1$.

6. (16 points) Let C be parameterized by $\mathbf{r}(t) = \langle \sqrt{2}\sin(t), 2\cos(t), \sqrt{2}\sin(t) \rangle$.

(a) Find a parameterization, $\ell(t)$, for the line tangent to C at $t = 0$.

(b) Find the TNB-frame for C.

(c) Compute the curvature of C.

7. (22 points) Consider the curve C parameterized by $\mathbf{r}(t) = \langle t^2, 3t, e^t \rangle$, $-3 \le t \le 10$.

(a) Compute the curvature of $\mathbf{r}(t)$.

(b) Compute the torsion of $\mathbf{r}(t)$.

(c) Set up the integral $\int_C (z + x\sin(y))\,ds$ [Obviously we cannot hope to evaluate this by hand – please don't try.]

(d) Compute the tangential and normal components of acceleration of $\mathbf{r}(t)$.

(e) Does this curve lie in a plane? Why or why not?