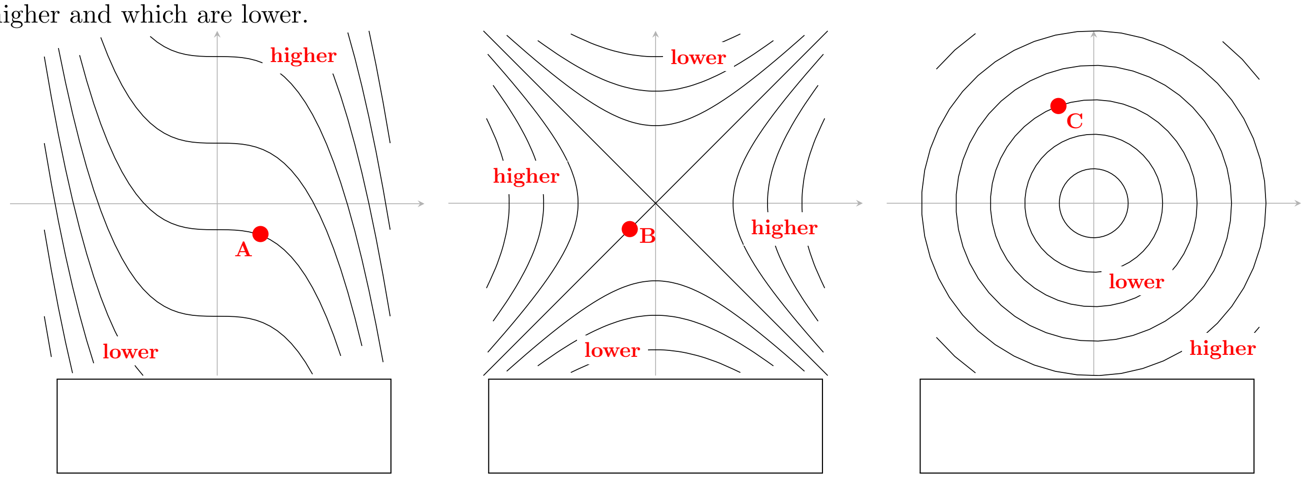
Math 2130 # Test #2 **March 16th, 2021**

Name: ________________________________ **Be sure to show your work!**

If $F(x,y) = C$, then $\dfrac{dy}{dx} = -\dfrac{F_x}{F_y}$ If $F(x,y,z) = C$, then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z}$

1. (12 points) Three level curve plots are shown below. I have labeled the levels so you know which curves are higher and which are lower.



(a) The plots above correspond to 3 of the 5 functions listed here: $f(x,y) = 1 - x^2 - y^2$, $f(x,y) = \sqrt{x^2+y^2}$, $f(x,y) = 2x^3 + y$, $f(x,y) = 2x^2 + y$, and $f(x,y) = x^2 - y^2$. Write the correct formula below each plot.

(b) Sketch a gradient vector at the points A, B, and C. If the vector is **0** or does not exist, draw an "X" on the point. [Don't worry about having the correct length. I'm just looking for the correct direction.]

2. (7 points) Let $w = f(x,y,z)$, $x = g(t)$, $y = h(t)$ and $z = \ell(t)$. State the chain rule for the derivative of w with respect to t. Clearly distinguish between regular derivatives (i.e., d's) and partial derivatives (i.e., ∂'s).

3. (9 points) Consider some unknown function $f(x,y)$.

(a) It is possible to have a function where $f_{xy}(3,4) = 5$ and $f_{yx}(3,4) = 6$? **YES / NO**
If not, why not? If so, what does this tell us?

(b) If $\nabla f(x,y)$ exists, can I conclude that $f(x,y)$ is differentiable? **YES / NO**

(c) If $\nabla f(x,y)$ is continuous, can I conclude that f is continuous? **YES / NO**

4. (10 points) Limits and continuity.

(a) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2+3y^2+xy^2}{x^2+y^2}$ exists and find this limit.

(b) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{10xy}{x^2+y^2}$ does not exist.

5. (15 points) Let $F(x,y,z) = x^2 + e^{yz} + \sin(xy^3z^2) + yz^2$.

Note: All three parts use the same function and point.

(a) Find an equation for the plane tangent to $x^2 + e^{yz} + \sin(xy^3z^2) + yz^2 = 2$ at $(x,y,z) = (1,0,2)$

(b) Find the directional derivative of F at the point $(1,0,2)$ in the direction of the vector $\langle -1, 2, -2\rangle$.

(c) Is it possible to find a direction vector $\mathbf{u}$ so that $D_{\mathbf{u}}F(1,0,2) = -2$? Why or why not?

6. (8 points) Suppose that $z = x^3y$ where x is measured within 1% accuracy and y is measured within 2% accuracy. Use a total derivative to estimate the maximum percent error in the corresponding z measurement.

7. (13 points) Let $f(x, y) = -x^3 + 12x + y^3 - 3y$.

(a) Compute the gradient and Hessian matrix for f.

(b) Find the quadratic approximation of f at $(x, y) = (-1, 2)$.

(c) Find and classify all of the critical points of f. [Use the "2^{nd}-derivative" test to determine if critical points are relative max's, min's or saddle points.]

8. (14 points) Suppose $f(x, y)$ is a "nice" function (with continuous partials of all orders).

(a) $Q(x, y) = 13 + 4(x - 3)^2 - 3(x - 3)(y + 2) + 2(y + 2)^2$ is the quadratic approx. at $(x, y) = (3, -2)$.

$\nabla f(3, -2) = \Big\langle \qquad\qquad\qquad\qquad \Big\rangle$ $\qquad H_f(3, -2) = \Big[\qquad\qquad\qquad\qquad \Big]$

Is $(x, y) = (3, -2)$ a critical point of $f(x, y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

(b) $Q(x, y) = 11 + 5(x + 1) - y + 4(x + 1)^2 - 6(x + 1)y$ is the quadratic approx. at $(x, y) = (-1, 0)$.

$\nabla f(-1, 0) = \Big\langle \qquad\qquad\qquad\qquad \Big\rangle$ $\qquad H_f(-1, 0) = \Big[\qquad\qquad\qquad\qquad \Big]$

Is $(x, y) = (-1, 0)$ a critical point of $f(x, y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

The linearization of $f(x, y)$ at $(x, y) = (-1, 0)$ is $L(x, y) =$ ______________________________

[If there is not enough information answer "N/A".]

9. (12 points) Use the method of Lagrange multipliers to find the minimum and maximum values of $f(x, y) = xy^2$ constrained to $x^2 + y^2 = 12$.