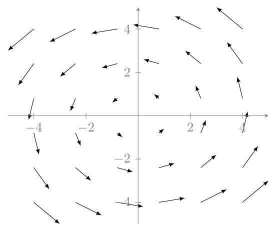
Math 2130 **Test #3** **April 14^{th}, 2021**

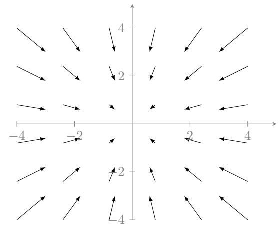
Name: ______________________________ **Be sure to show your work!**

$$x = \rho\cos(\theta)\sin(\varphi) \qquad J = \rho^2\sin(\varphi)$$
$$y = \rho\sin(\theta)\sin(\varphi)$$
$$z = \rho\cos(\varphi) \qquad \cos^2(\theta) = \frac{1}{2}\left(1+\cos(2\theta)\right)$$

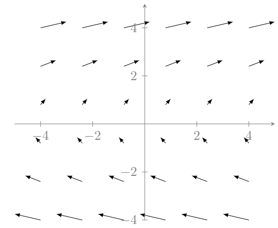
1. (12 points) The following are plots of several vector fields. Please note that all of the vectors have been scaled down so they do not overlap each other. Write A, B, and C next to the appropriate vector field's formula. Put an X next to the formula whose vector field is **not** shown. Also, for each vector field is **F** conservative? Circle "Yes" or "No".



A



B



C

☐ $\mathbf{F}(x,y) = \langle y, 1\rangle$ Yes / No

☐ $\mathbf{F}(x,y) = \langle x, y\rangle$ Yes / No

☐ $\mathbf{F}(x,y) = \langle -x, -y\rangle$ Yes / No

☐ $\mathbf{F}(x,y) = \langle -y, x\rangle$ Yes / No

2. (10 points) Use a double Riemann sum to approximate $\iint_R y\cos(x^2+1)\,dA$ where $R = [-8,8]\times[0,4]$.

Use midpoint rule and a 2×2 grid of rectangles (2 across and 2 up) to partition R. (Don't worry about simplifying.)

3. (7 points) Let $\mathbf{F}(x,y,z) = \langle 10, xyz, e^x\rangle$ and C be the curve parameterized by $\mathbf{r}(t) = \left\langle t^2, t^4, t^6\right\rangle$ where $-7 \le t \le 9$.

Set up but **do not** evaluate the line integral: $\int_C \mathbf{F}\bullet d\mathbf{r}$.

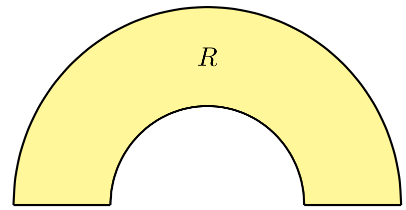
Centroid: $m = \iint_R 1\,dA$, $M_y = \iint_R x\,dA$, $M_x = \iint_R y\,dA$, $(\bar{x}, \bar{y}) = (M_y/m, M_x/m)$

4. (13 points) Let R be the annular region between $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ where $y \geq 0$ (pictured below).
[*Warning:* One of the integrals below will have to be split into *several* pieces.]

(a) Set up the integral $\iint_R y\,dA$ in polar coordinates.

(b) Set up the integral $\iint_R y\,dA$ rectangular coordinates.

(c) Find the centroid of R.



5. (11 points) Consider $\int_0^2 \int_{3x}^6 \sin(y^2)\,dy\,dx$.

(a) Sketch the corresponding region of integration.

(b) Compute the iterated integral. *Hint:* You cannot integrate $\int \sin(y^2)\,dy$ in terms of elementary functions.

6. (11 points) **Set up** the integral $\iint\limits_R e^{(x-y)^2}\,dA$ where R is the region bounded by the lines $y = x - 3$, $y = x - 4$, $y = -2x$, and $x = 0$. *Note:* Use the change of coordinates: $u = 2x + y$ and $v = x - y$. **DO NOT** evaluate this integral.

7. (10 points) Let E be the region bounded by $z = x^2 + y^2 + 1$ and $z = 5$. Compute the integral $\iiint_E \frac{1}{\sqrt{x^2+y^2}}\,dV$.

8. (12 points) Consider the integral: $I = \int_{-7}^{0} \int_{-\sqrt{49-x^2}}^{\sqrt{49-x^2}} \int_{-\sqrt{49-x^2-y^2}}^{0} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx.$

(a) Rewrite I in the following order of integration: $\iiint \quad dy\, dx\, dz$.
Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.
Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.
Do **not** evaluate the integral.

9. (14 points) Let E be the region above the cone $z = 2\sqrt{x^2+y^2}$, below $z = 18$, and where $y \geq 0$.

(a) Set up $\iiint_E x + z\, dV$ in rectangular coordinates.

(b) Set up $\iiint_E x + z\, dV$ in cylindrical coordinates.

(c) Set up $\iiint_E x + z\, dV$ in spherical coordinates.