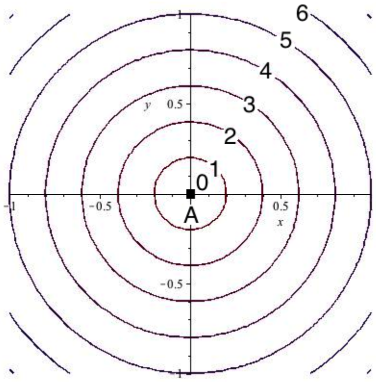
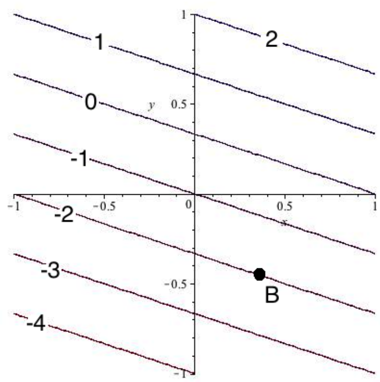
Math 2130-101 **Test #2** **July 22^{nd}, 2014**

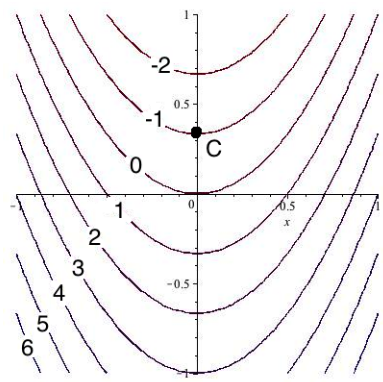
Name: ______________________________ **Be sure to show your work!**

If $F(x,y)=C$, then $\dfrac{dy}{dx}=-\dfrac{F_x}{F_y}$ If $F(x,y,z)=C$, then $\dfrac{\partial z}{\partial x}=-\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y}=-\dfrac{F_y}{F_z}$

1. (11 points) Three level curve plots are shown below. I have labeled the levels so you know which curves are higher and which are lower.







(a) The plots above correspond to 3 of the functions listed here: $f(x,y)=9-x^2-y^2$, $f(x,y)=x+3y-1$, $f(x,y)=5\sqrt{x^2+y^2}$, $f(x,y)=4x^2-3y$, and $f(x,y)=4y^2-3x$. Write the correct formula below each plot.

(b) Sketch a gradient vector at the points A, B, and C. If the vector is $\mathbf{0}$, draw an "X" on the point.
[Don't worry about having the correct length. I'm just looking for the correct direction.]

(c) If A, B, or C is a critical point, write what kind of point it is (i.e. min, max, saddle, or other).

2. (8 points) Let $z=f(x,y)$ where $x=r\cos(\theta)$ and $y=r\sin(\theta)$.

Show that $\left(\dfrac{\partial z}{\partial x}\right)^2+\left(\dfrac{\partial z}{\partial y}\right)^2=\left(\dfrac{\partial z}{\partial r}\right)^2+\dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial \theta}\right)^2.$

3. (10 points) Let $x^3y^2 + e^{xyz} + z^2 = 2$.

(a) Find an equation for the plane tangent to the above surface at the point $(x, y, z) = (0, 1, -1)$.

(b) Considering z as a variable depending on x and y (defined implicitly above), find $\dfrac{\partial z}{\partial x}$.

4. (10 points) Limits

(a) Show the following limit **does** exist: $$\lim_{(x,y)\to(0,0)} \frac{x^2 + xy + y^2}{\sqrt{x^2 + y^2}}$$

(b) Show the following limit **does not** exist: $$\lim_{(x,y)\to(0,0)} \frac{x^2 + xy}{2x^2 + y^2}$$

5. (9 points) Suppose we have a function of two variables: $f(x, y)$.

(a) Suppose that $f_{xy}(1, 2) = 3$ and $f_{yx}(1, 2) = 4$. What (if anything) can I conclude about f or its partials?

(b) Suppose f_x and f_y exist everywhere. Can I conclude f is a continuous function? **YES / NO**

(c) Suppose f_x and f_y are continuous everywhere. Can I conclude that f is differentiable? **YES / NO**

6. (12 points) Let $f(x, y) = -x^4 + 4xy - 2y^2 - 3$.

(a) Find the gradient of f and the Hessian matrix of f.

(b) Find the quadratic approximation of f at $(x, y) = (0, -1)$.

(c) Find an classify the critical point(s) of $f(x, y)$.
[Use the "2^{nd}-derivative" test to determine if critical points are relative max's, min's or saddle points.]

7. (8 points) Let $z = \dfrac{y}{x^2}$. Use a differential (i.e. total derivative) to estimate the maximal change in z if we start at $(x, y) = (-1, 2)$ and then x changes by ± 0.2 and y changes by ± 0.1.

8. (10 points) A Directed Problem. [Assume that the function $g(x, y)$ in parts (b) and (c) is differentiable.]

(a) Let $f(x, y, z) = e^{xy^2} + y^2 z$. Find the directional derivative of f at the point $(x, y) = (0, -1, 3)$ and in the same direction as $\mathbf{v} = \langle -2, 1, 3 \rangle$.

(b) Suppose that $\nabla g(1, 2) = \langle 3, 4 \rangle$. What is the maximum possible value of $D_{\mathbf{u}} g(1, 2)$? Give a unit vector which causes this maximum to occur.

(c) Again, suppose $\nabla g(1, 2) = \langle 3, 4 \rangle$. Mr. Pete claims that for some unit vector $\mathbf{u}$ he found that $D_{\mathbf{u}} g(1, 2) = -10$. Why must Mr. Pete be wrong?

9. (10 points) Suppose $f(x, y)$ is a "nice" function (with continuous partials of all orders).

(a) $Q(x, y) = 1 + (x + 3)^2 + 4(x + 3)(y - 5) + 4(y - 5)^2$ is the quadratic approx. at $(x, y) = (-3, 5)$.

$\nabla f(-3, 5) = \Big\langle \qquad\qquad\qquad\qquad \Big\rangle$ $\qquad H_f(-3, 5) = \Bigg[\qquad\qquad\qquad\qquad \Bigg]$

Is $(x, y) = (-3, 5)$ a critical point of $f(x, y)$? **YES / NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

(b) $Q(x, y) = 2x - 3x^2 + 4x(y - 5) - 6(y - 5)^2$ is the quadratic approx. at $(x, y) = (0, 5)$.

$\nabla f(0, 5) = \Big\langle \qquad\qquad\qquad\qquad \Big\rangle$ $\qquad H_f(0, 5) = \Bigg[\qquad\qquad\qquad\qquad \Bigg]$

Is $(x, y) = (0, 5)$ a critical point of $f(x, y)$? **YES / NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

10. (12 points) Use the method of Lagrange multipliers to find the minimum and maximum values of $f(x, y, z) = xyz$ constrained to $x^2 + 2y^2 + z^2 = 12$.