Math 2130-101 **Test #1** **July 8th, 2015**

Name: ______________________________ **Be sure to show your work!**

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{\mathbf{u}\bullet\mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \qquad \mathbf{r}''(t) = \left(\frac{\mathbf{r}'(t)\bullet\mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t) \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$m = \int_C \rho\, ds \qquad (\bar{x},\bar{y},\bar{z}) = \frac{1}{m}\left(\int_C x\rho\, ds, \int_C y\rho\, ds, \int_C z\rho\, ds\right) \qquad \kappa = \frac{|f''(x)|}{\left(1+(f'(x))^2\right)^{\frac{3}{2}}}$$

1. (17 points) Vector Basics: Let $\mathbf{u} = \langle 2,1,-1\rangle$, $\mathbf{v} = \langle 3,1,2\rangle$, and $\mathbf{w} = \langle -1,2,-2\rangle$.

(a) Compute the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$.

(b) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

(c) Find two vectors of length 5 which are parallel to $\mathbf{w}$.

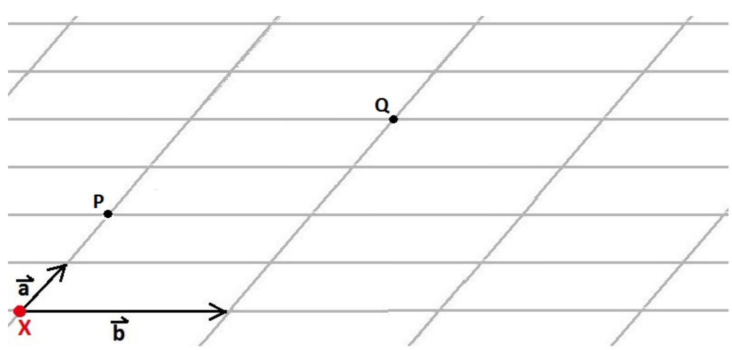
(d) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trig. functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(e) Match the statement on the left to the corresponding statement on the right...

☐	$(\mathbf{a}\times\mathbf{b})\bullet\mathbf{b} = 0$	**D)** $\mathbf{a}$ and $\mathbf{b}$ are parallel
☐	$(\mathbf{a}\bullet\mathbf{b})\times(\mathbf{a}\bullet\mathbf{b}) = \mathbf{0}$	**B)** $\mathbf{a}$ and $\mathbf{b}$ are orthogonal
☐	$\mathbf{a}\times\mathbf{b} = \mathbf{0}$	**A)** is always true
☐	$\mathbf{a}\bullet\mathbf{b} = 0$	**C)** is nonsense

(f) The vectors $\mathbf{a}$ and $\mathbf{b}$ are shown to the right. They are based at the point X. Sketch the vector $\mathbf{b}-\mathbf{a}$ based at the point P and sketch the vector $2\mathbf{a}+\mathbf{b}$ based at the point Q.



2. (10 points) Let ℓ_1 be parametrized by $\mathbf{r}_1(t) = \langle 3, -1, -1\rangle + \langle 1, -1, 1\rangle t$ and let ℓ_2 be the line which passes through the points $P = (2, 1, -1)$ and $Q = (3, 1, 1)$. Determine if ℓ_1 and ℓ_2 are... (circle the correct answer)

the same, parallel (but not the same), intersecting, or skew.

3. (14 points) Plane old geometry.

(a) Find the (scalar) equation of the plane through the points $A = (2, 1, 0)$, $B = (3, 3, 1)$, and $C = (2, 3, -1)$.

(b) Find the area of the triangle with vertices A, B, and C (from part (a)).

(c) The planes: $3x - 2y + 2z = 10$ and $2x - y - 4z = 7$ are... [Circle **ALL** that apply.]

parallel perpendicular intersecting the same

4. (8 points) Is the curvature of $y = e^{-2x}$ ever zero? Yes / No

5. (12 points) Parameterization, arc length, and a line integral.

(a) Let C be the upper-half of the circle $x^2 + y^2 = 4$. Parameterize C and then compute its centroid. [*Hint:* Take advantage of geometry and symmetry.]

6. (15 points) Let C be parameterized by $\mathbf{r}(t) = \langle \sin(t), t^3, e^t \rangle$ where $-3 \leq t \leq 1$.

(a) Set up the line integral $\int_C (x^2 e^y + 4z)\, ds$. [Do not try to evaluate this integral. It will only end in tears.]

(b) Find the curvature of $\mathbf{r}(t)$.

(c) Find the tangential and normal components of acceleration for $\mathbf{r}(t)$.

7. (12 points) Find the TNB-frame for $\mathbf{r}(t) = \langle 4\sin(t), 4\cos(t), 3t\rangle$.

Does this curve lie in a plane? Why or why not?

8. (12 points) No numbers here. Choose **ONE** of the following:

I. Suppose that $|\mathbf{r}(t)| = c$ (the length of $\mathbf{r}(t)$ is constant). Prove that $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are orthogonal.

II. Suppose that $\mathbf{a}$ and $\mathbf{b}$ are unit vectors. Prove that $(\mathbf{a} \bullet \mathbf{b})^2 + |\mathbf{a} \times \mathbf{b}|^2 = 1$. [*Suggestion:* Use a fundamental identity for both $\mathbf{a} \bullet \mathbf{b}$ and $|\mathbf{a} \times \mathbf{b}|$. Don't try to prove this with components.]