Math 2130 **Test #1** **July 11^{th}, 2016**

Name: ______________________________ **Be sure to show your work!**

$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \dfrac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}$ $\quad \mathbf{r}''(t) = \left(\dfrac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t)$ $\quad \kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$

$m = \displaystyle\int_C \rho\, ds$ $\quad (\bar{x}, \bar{y}, \bar{z}) = \dfrac{1}{m}\left(\displaystyle\int_C x\rho\, ds, \int_C y\rho\, ds, \int_C z\rho\, ds\right)$ $\quad \kappa = \dfrac{|f''(x)|}{\left(1 + (f'(x))^2\right)^{\frac{3}{2}}}$

1. (20 points) Vector Basics: Let $\mathbf{v} = \langle 3, -1, 2 \rangle$ and $\mathbf{w} = \langle 2, 1, -1 \rangle$.

(a) Find a **unit** vector perpendicular to both $\mathbf{v}$ and $\mathbf{w}$.

(b) Compute $\text{proj}_{\mathbf{v}}(\mathbf{w})$

(c) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trig. functions).

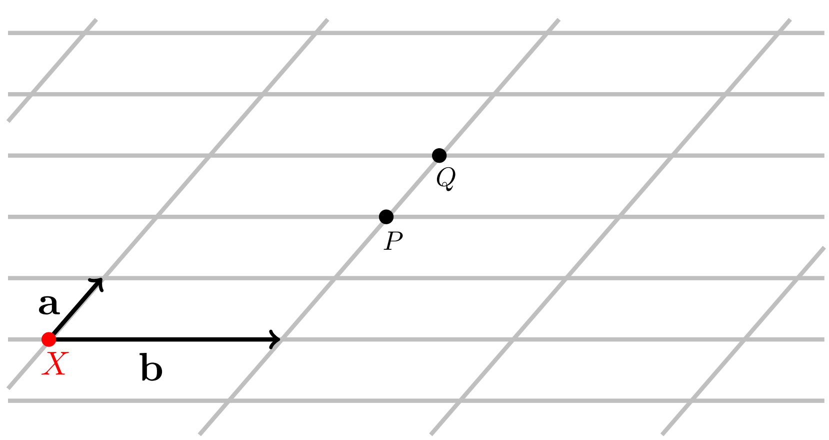
Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

(d) Fill in the blanks ($\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors)...

(i) $|\mathbf{a} \times \mathbf{b}|$ computes the ____________________ of the ________________________ spanned by $\mathbf{a}$ and $\mathbf{b}$.

(ii) $|\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c})|$ computes the ____________________ of the ________________________ spanned by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.

(e) The vectors $\mathbf{a}$ and $\mathbf{b}$ are shown to the right. They are based at the point X. Sketch the vector $\mathbf{a} - \mathbf{b}$ based at the point P and sketch the vector $2\mathbf{a} + \mathbf{b}$ based at the point Q.



2. (10 points) Let ℓ_1 be parametrized by $\mathbf{r}_1(t) = \langle 2t+1, -t+3, t\rangle$ and let ℓ_2 be the line which passes through the points $P = (2, 1, -1)$ and $Q = (-2, 3, -3)$. Determine if ℓ_1 and ℓ_2 are... (circle the correct answer)

the same, parallel (but not the same), intersecting, or skew.

3. (13 points) Plane old geometry.

(a) Find a (scalar) equation for the plane that containing the line $\mathbf{r}(t) = \langle 1 + 2t, -1 - t, -2t\rangle$ and the point $P = (1, 0, -2)$.

(b) Consider the two planes: $x + 2y + 2z + 4 = 0$ and $2x - y - 2z + 5 = 0$. If these are parallel planes explain why they are parallel. If these planes intersect, find the angle between these planes (don't worry about evaluating inverse trig. functions).

4. (8 points) Find the area of the triangle with vertices $A = (1, 0, 2)$, $B = (2, 2, 5)$, and $C = (1, 3, 3)$.

5. (8 points) Compute the curvature of $\mathbf{r}(t) = \langle t^2, \sin(t), e^{2t} \rangle$

6. (13 points) Consider the circle C: $(x-a)^2 + (y-b)^2 = R^2$ (where $R > 0$).

(a) Parameterize C and compute its arc length using your parameterization.

(b) Compute the curvature of C using your parameterization.

7. (16 points) Consider the curve parameterized by $\mathbf{r}(t) = \langle 3\sin(t), 4t, -3\cos(t)\rangle$ where $0 \leq t \leq 2\pi$.

(a) Compute the centroid of this curve.

(b) Find the TNB-frame for $\mathbf{r}(t)$.

Does this curve lie in a plane? Why or why not?

8. (12 points) Choose **ONE** of the following: [In both cases, drawing a good explanatory picture will earn you some partial credit – but for full credit you need more.]

I. Let $\mathbf{a}$ and $\mathbf{b}$ be vectors. Simplify $(2\mathbf{a} - \mathbf{b}) \bullet (\mathbf{a} \times \mathbf{b})$. Explain your answer geometrically.

II. Let C be the curve in the plane defined by $y = f(x)$ (where f is a function with at least two continuous derivatives). Use the special formula for curvature to explain why $\kappa(x) = 0$ exactly when C is a line (or part of a line).