Math 2130 | # Test #3 | **July 27^{th}, 2016**

Name: ______________________________ **Be sure to show your work!**

$$\begin{aligned} x &= \rho\cos(\theta)\sin(\varphi) \\ y &= \rho\sin(\theta)\sin(\varphi) \\ z &= \rho\cos(\varphi) \end{aligned}$$

$J = \rho^2\sin(\varphi)$

$\cos^2(\theta) = \frac{1}{2}\left(1+\cos(2\theta)\right)$

1. (12 points) Let R be the region bounded by $y = 5 - x^2$ and $y = -x + 3$.

[Warning: One of the integrals below will have to be split into 2 pieces.]

(a) Sketch the region R.

(b) Set up the integral $\iint_R f(x,y)\,dA$ using the order of integration "$dy\,dx$". [*Note:* You cannot evaluate this integral.]

(c) Set up the integral $\iint_R f(x,y)\,dA$ using the order of integration "$dx\,dy$". [*Note:* You cannot evaluate this integral.]

2. (12 points) Evaluate $\displaystyle\int_0^4\int_{\sqrt{x}}^2 \sin(y^3)\,dy\,dx$.

Note/Hint: You cannot integrate $\int \sin(y^3)\,dy$ in terms of elementary functions.

3. (12 points) Let R be the left half (i.e. $x \leq 0$) of the annulus lying between $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Sketch a picture of R then compute its centroid. You should be able to use symmetry and geometry to cut down the amount of integration you need to do.

$$m = \iint_R 1\, dA \qquad M_y = \iint_R x\, dA \qquad M_x = \iint_R y\, dA \qquad (\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

4. (13 points) Compute $\displaystyle\iint_R (x+y)e^{x-y}\, dA$ where R is the region bounded by $x + y = 1$, $x - y = 0$ and $y = 0$.

Use a (natural) change of coordinates which simplifies the region R and... don't forget the Jacobian!

5. (15 points) Let E be the region bounded by $z = 5\sqrt{x^2 + y^2}$ and $z = 10$. Set up integrals which computes the volume of E using the following orders of integration: [Do **not** evaluate these integrals.]

(a) $\int_?^? \int_?^? \int_?^? ???\, dz\, dy\, dx$

(b) $\int_?^? \int_?^? \int_?^? ???\, dx\, dz\, dy$

(c) Set up this integral in cylindrical coordinates.

(d) Set up this integral in spherical coordinates.

6. (12 points) Compute $\iiint_E \frac{1}{\sqrt{x^2 + y^2 + z^2}}\, dV$ where E is the solid region outside $x^2+y^2+z^2 = 1$, inside $x^2+y^2+z^2 = 4$, and in the first octant.

7. (12 points) Consider the region E bounded below by $z = x^2 + y^2$ and above by $z = 4$. Compute the centroid of E.

Free information: The volume of E is 8π.

$$m = \iiint_E 1\,dV \qquad M_{yz} = \iiint_E x\,dV \qquad M_{xz} = \iiint_E y\,dV \qquad M_{xy} = \iiint_E z\,dV \qquad (\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m}\right)$$

8. (12 points) Consider the integral: $I = \displaystyle\int_0^7 \int_{-\sqrt{49-x^2}}^0 \int_{-\sqrt{49-x^2-y^2}}^0 z\cos(x^2+y^2+z^2)\,dz\,dy\,dx.$

(a) Rewrite I in the following order of integration: $\iiint \quad dy\,dz\,dx.$

Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.

Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.

Do **not** evaluate the integral.