Math 2130 — **Final Exam** — **June 28^{th}, 2019**

Name: ____________________________________ **Be sure to show your work!**

1. (7 points) Let $\mathbf{F}(x,y,z) = \langle x^2 + yz, y + e^z, \sin(xy^2)\rangle$. Compute $\nabla \times \mathbf{F}$ and $\nabla \bullet \mathbf{F}$.

If $\mathbf{F}$ conservative? Yes / No

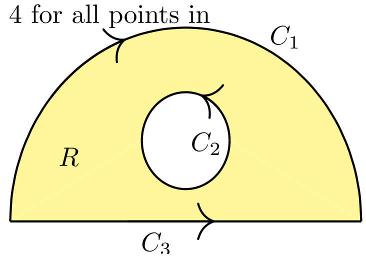
2. (11 points) Let $\mathbf{F}(x,y,z) = \left\langle 2xyz + 1, x^2z + z, x^2y + y + 2z \right\rangle$. This is a conservative vector field (I've checked for you).

(a) Use the fundamental theorem of line integrals to compute $\int_C \mathbf{F} \bullet d\mathbf{r}$ where C is the line segment from $(1,-1,0)$ to $(1,2,2)$.

(b) Recompute $\int_C \mathbf{F} \bullet d\mathbf{r}$ directly (i.e. parameterize C etc.).

3. (10 points) Let C be the boundary of a triangle with vertices $(0,-1)$, $(2,-1)$, and $(2,3)$ oriented counter-clockwise. Compute $\int_C \left(\cos(x^6 + e^{3x}) + y\right)\, dx + \left(x^2 + \ln(y^8 + 3)\right)\, dy$.

4. (9 points) C_1 is an upper-half of a circle of radius 3 (oriented clockwise), C_2 is a circle of radius 1 (oriented counter clockwise), and C_3 is a line segment closing off the semi-circle (oriented left to right). Let $\mathbf{F}(x,y) = \langle M(x,y), N(x,y)\rangle$ be a vector field such that M and N have continuous first partials and in addition, $N_x - M_y = 4$ for all points in region R. We also know $\int_{C_2} \mathbf{F} \bullet d\mathbf{r} = 2\pi$ and $\int_{C_3} \mathbf{F} \bullet d\mathbf{r} = \pi$. *Note:* The area of R is $7\pi/2$.



Compute $\int_{C_1} M(x,y)\, dx + N(x,y)\, dy \quad = \quad$ ____________________.

5. (12 points) Let S_1 be the lower hemisphere: $x^2 + y^2 + z^2 = 1$, $z \leq 0$ and S_2 be the disk $x^2 + y^2 \leq 1$ in the xy-plane. Orient both S_1 and S_2 upward. Let $\mathbf{F}$ be a smooth vector field such that $\iint_{S_1} \mathbf{F} \bullet \mathbf{n}\, d\sigma = 10\pi$ and $\nabla \bullet \mathbf{F} = 5x^2 + 5y^2 + 5z^2$. Find $\iint_{S_2} \mathbf{F} \bullet \mathbf{n}\, d\sigma$.

6. (12 points) Find the centroid of the part of the cone $z = 3\sqrt{x^2+y^2}$ where $z \leq 6$.

Note: This is a **surface**. You should be computing **surface integrals**.

$$m = \iint_{S_1} 1\,d\sigma \qquad M_{yz} = \iint_{S_1} x\,d\sigma \qquad M_{xz} = \iint_{S_1} y\,d\sigma \qquad M_{xy} = \iint_{S_1} z\,d\sigma$$

7. (13 points) Let S_1 be the surface parameterized by $\mathbf{r}(u,v) = \langle u\cos(v), u\sin(v), 9-u^2\rangle$ where $1 \leq u \leq 2$ and $\pi \leq v \leq 2\pi$.

(a) Find both orientations for S_1.

(b) Set up but **do not evaluate** the surface integral $\iint_{S_1} z\ln(x^2+y^2+1)\,d\sigma$. [Don't worry about simplifying.]

(c) Set up but **do not evaluate** the flux integral $\iint_{S_1} \mathbf{F} \bullet \mathbf{n}\,d\sigma$ where S_1 is <u>oriented downward</u> and $\mathbf{F}(x,y,z) = \langle xz, y, x+y\rangle$.
[Don't worry about computing the dot product or any significant simplification.]

8. (11 points) Let S_1 be the surface of the solid cylinder defined by $x^2 + y^2 \leq 4$ and $-2 \leq z \leq 3$. Orient S_1 outward and let $\mathbf{F}(x, y, z) = \left\langle 4xy^2 + \cos(y^{15} + z), \ln(x^2 + z^4 + 1) + 2y, 4x^2z + e^{x^3y} \right\rangle$. Compute the flux integral $\iint_{S_1} \mathbf{F} \bullet \mathbf{n}\, d\sigma$.

9. (15 points) Let S_1 be the upper hemisphere: $x^2 + y^2 + z^2 = 4$ and $z \geq 0$. Orient S_1 downward.

Verify Stokes' Theorem for the surface S_1, its boundary, and the vector field $\mathbf{F}(x, y, z) = \langle z, x, x \rangle$.