Math 2130-101 # Test #1 **June 5^{th}, 2019**

Name: ______________________________ **Be sure to show your work!**

$\text{proj}_{\mathbf{w}}(\mathbf{v}) = \dfrac{\mathbf{v} \bullet \mathbf{w}}{|\mathbf{w}|^2}\mathbf{w}$ $\quad$ $\mathbf{r}''(t) = \left(\dfrac{\mathbf{r}'(t) \bullet \mathbf{r}''(t)}{|\mathbf{r}'(t)|}\right)\mathbf{T}(t) + \left(\dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}\right)\mathbf{N}(t)$ $\quad$ $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$

$m = \displaystyle\int_C \delta(x,y,z)\,ds$ $\quad$ $(\bar{x},\bar{y},\bar{z}) = \dfrac{1}{m}\left(\displaystyle\int_C x\delta(x,y,z)\,ds, \int_C y\delta(x,y,z)\,ds, \int_C z\delta(x,y,z)\,ds\right)$ $\quad$ $\tau = \dfrac{(\mathbf{r}'(t) \times \mathbf{r}''(t)) \bullet \mathbf{r}'''(t)}{|\mathbf{r}'(t) \times \mathbf{r}''(t)|^2}$

1. (24 points) Vector Basics: Let $\mathbf{v} = \langle -2, 0, 1\rangle$ and $\mathbf{w} = \langle 1, 2, -2\rangle$.

(a) Compute $\text{proj}_{\mathbf{w}}(\mathbf{v})$.

(b) Find two *unit* vectors which are *orthogonal* (i.e., perpendicular) to both $\mathbf{v}$ and $\mathbf{w}$.

(c) Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (don't worry about evaluating inverse trig. functions).

Is this angle... **right,** **acute,** or **obtuse** ? (Circle your answer.)

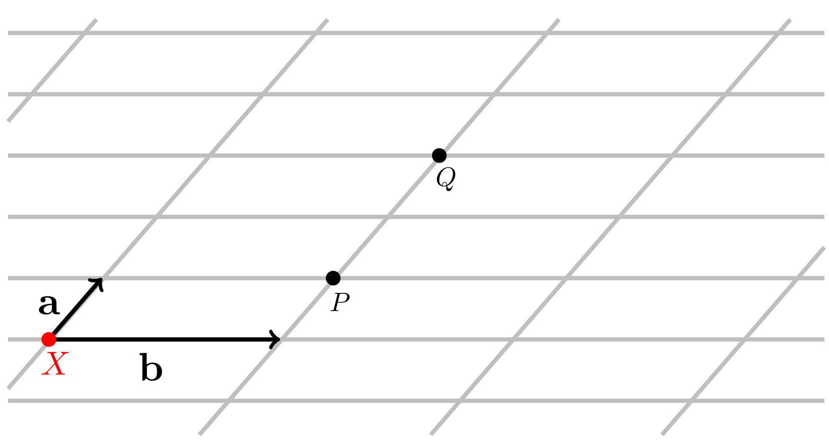
(d) Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be vectors in $\mathbb{R}^3$. Fill in the blanks:

If $\mathbf{a} \times \mathbf{b} = \mathbf{0}$, then $\mathbf{a}$ and $\mathbf{b}$ are ______________________.

If $\mathbf{a} \bullet \mathbf{b} = 0$, then $\mathbf{a}$ and $\mathbf{b}$ are ______________________.

If $(\mathbf{a} \times \mathbf{b}) \bullet \mathbf{c} = 0$, then $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are ______________________.

(e) The vectors $\mathbf{a}$ and $\mathbf{b}$ are shown to the right. They are based at the point X. Sketch the vector $2\mathbf{a} - \mathbf{b}$ based at the point P and sketch the vector $-\mathbf{a} + \mathbf{b}$ based at the point Q.



2. (14 points) Give vector valued functions which parameterize the following curves.

Don't forget to specify a domain for your parameter.

(a) The line segment from $A = (1, 2, 0)$ to $B = (4, 0, 1)$.

(b) The circle $(x + 1)^2 + (y - 3)^2 = 25$.

3. (14 points) Lines and Planes

(a) Find an equation for the plane which passes through the points $A = (1, 0, -1)$, $B = (2, 3, 0)$, and $C = (3, -1, 1)$.

(b) Consider the line parameterized by $\mathbf{r}(t) = \langle 2t, -4t + 1, 6t - 2 \rangle$ and the plane $-x + 2y - 3z = 10$.

The line and plane are (circle **all** that apply)... parallel perpendicular intersecting.

4. (12 points) Suppose that a particle's velocity is given by $\mathbf{v}(t) = t^2\mathbf{i} + 5e^t\mathbf{j}$. In addition, we have that this particle's initial position is $\mathbf{r}_0 = \mathbf{i} + 2\mathbf{j}$. [For what it's worth, measurements are made in meters and seconds.]

(a) The particle's initial speed is _______ meters per second.

(b) Find the particle's acceleration $\mathbf{a}(t)$.

(c) Find the particle's position function $\mathbf{r}(t)$.

5. (18 points) Let C be the curve parameterized by $\mathbf{r}(t) = \langle 3t, 4\sin(t), 4\cos(t)\rangle$, $-\pi \leq t \leq 3\pi$.

(a) Compute the **TNB**-frame of C.

(b) Compute the curvature of C.

(c) Compute the arc length of C.

(d) Set up (but **do not evaluate**) the line integral $\int_C x\, e^{y^2+z^2}\, ds$.

6. (18 points) Consider the curve C parameterized by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$.

(a) Find a parameterization for the line tangent to C at $t = 2$.

(b) Compute the curvature of C.

(c) Compute the torsion of C.

(d) Compute the tangential and normal components of acceleration of $\mathbf{r}(t)$.

(e) Does this curve lie in a plane? Why or why not?