Math 2130

Test #2

June 17th, 2019

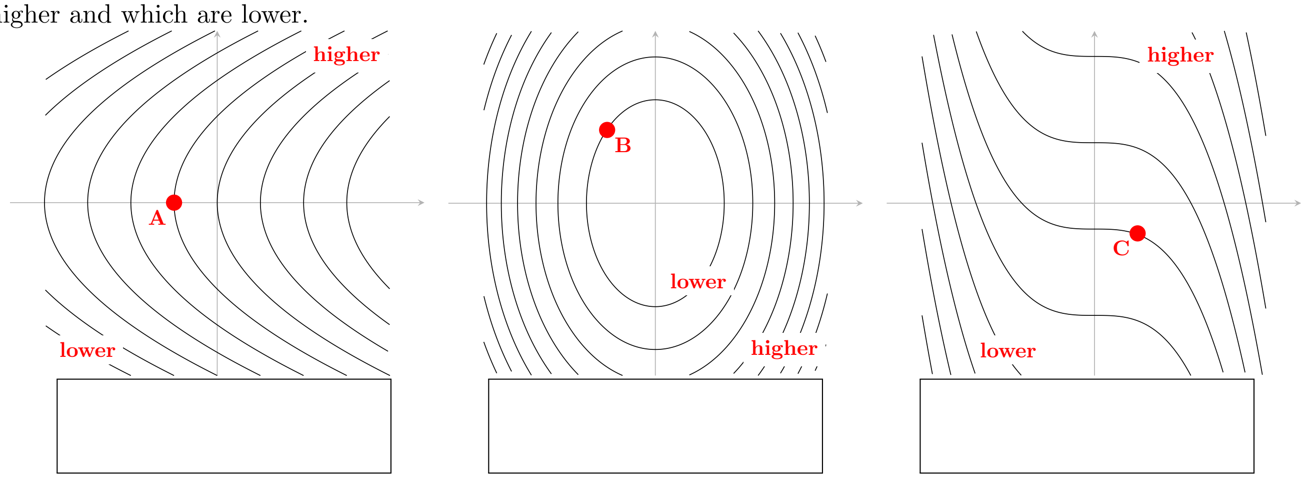
Name: ______________________________

Be sure to show your work!

If $F(x,y)=C$, then $\dfrac{dy}{dx}=-\dfrac{F_x}{F_y}$

If $F(x,y,z)=C$, then $\dfrac{\partial z}{\partial x}=-\dfrac{F_x}{F_z}$ and $\dfrac{\partial z}{\partial y}=-\dfrac{F_y}{F_z}$

1. (12 points) Three level curve plots are shown below. I have labeled the levels so you know which curves are higher and which are lower.



(a) The plots above correspond to 3 of the 5 functions listed here: $f(x,y)=x^2/2+y^2/3$, $f(x,y)=x^2+y^2$, $f(x,y)=2x^3+y$, $f(x,y)=y-x^2$, and $f(x,y)=x-y^2$. Write the correct formula below each plot.

(b) Sketch a gradient vector at the points A, B, and C. If the vector is **0** or does not exist, draw an "X" on the point. [Don't worry about having the correct length. I'm just looking for the correct direction.]

2. (8 points) Let $w=f(x,y,z)$, $x=g(u,v)$, $y=h(u,v)$, and $z=\ell(u,v)$. State the chain rule for $\dfrac{\partial w}{\partial u}$.

3. (9 points) Suppose we have a function $f(x,y)$ where $\nabla f(x,y)$ exists everywhere.

(a) It is possible for $f_{xy}(2,3)=4$ and $f_{yx}(2,3)=5$? If not, why not? If so, what does this tell us?

(b) Can I conclude $f(x,y)$ is differentiable? **YES / NO**

(c) Can I conclude that $f(x,y)$ is continuous? **YES / NO**

4. (10 points) Limits and continuity.

(a) Where is the function $f(x, y) = \ln(x^2 + y^2)$ continuous?

(b) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2 + y^2}$ does not exist.

5. (14 points) Let $F(x, y, z) = y^2z^3 + e^{x^2z}$. *Note:* All three parts use the same function and point.

(a) Find an equation for the plane tangent to $y^2z^3 + e^{x^2z} = -3$ at $(x, y, z) = (0, 2, -1)$

(b) Find the directional derivative $D_{\mathbf{u}}F(0, 2, -1)$ where $\mathbf{u}$ points in the same direction as $\mathbf{v} = \langle 2, -2, 1\rangle$.

(c) Find the direction vector $\mathbf{u}$ which maximizes $D_{\mathbf{u}}F(0, 2, -1)$. What is the maximum value?

6. (8 points) Let $e^{3x}\sin(y^2 z) + y\ln(x^4 + z^2) = 99$. Assuming z is a function of x and y, find $\dfrac{\partial z}{\partial y}$.

[Don't worry about simplifying.]

7. (13 points) Let $f(x, y) = -x^4 + 4xy - 2y^2 - 3$.

(a) Compute the gradient and Hessian matrix for f.

(b) Find the quadratic approximation of f at $(x, y) = (0, -1)$.

(c) Find and classify all of the critical points of f. [Use the "2^{nd}-derivative" test to determine if critical points are relative max's, min's or saddle points.]

To speed you along: There are exactly 3 critical points. Their x-coordinates are $x = 0, \pm 1$.

8. (14 points) Suppose $f(x,y)$ is a "nice" function (with continuous partials of all orders).

(a) $Q(x,y) = 1 + 2(x-1) + 3(x-1)(y+2) + 4(y+2)^2$ is the quadratic approx. at $(x,y) = (1,-2)$.

$\nabla f(1,-2) = \Big\langle \qquad\qquad \Big\rangle \qquad H_f(1,-2) = \Big[\qquad\qquad \Big]$

Is $(x,y) = (1,-2)$ a critical point of $f(x,y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

The linearization of $f(x,y)$ at $(x,y) = (1,-2)$ is $L(x,y) =$ ____________________

[If there is not enough information answer "N/A".]

(b) $Q(x,y) = 12 - 5(x-2)^2 + (x-2)y - 3y^2$ is the quadratic approx. at $(x,y) = (2,0)$.

$\nabla f(2,0) = \Big\langle \qquad\qquad \Big\rangle \qquad H_f(2,0) = \Big[\qquad\qquad \Big]$

Is $(x,y) = (2,0)$ a critical point of $f(x,y)$? **YES** / **NO**

If not, why not? If so, what kind (relative min, relative max, saddle point or not enough information)?

9. (12 points) Use the method of Lagrange multipliers to find the minimum and maximum values of $f(x,y) = 2x - 4y$ constrained to $x^2 + y^2 = 5$.