Math 2130 | # Test #3 | **June** 24^{th}**, 2019**

Name: ______________________________ **Be sure to show your work!**

$$x = \rho\cos(\theta)\sin(\varphi)$$
$$y = \rho\sin(\theta)\sin(\varphi)$$
$$z = \rho\cos(\varphi)$$

$$J = \rho^2\sin(\varphi)$$

$$\cos^2(\theta) = \tfrac{1}{2}\left(1+\cos(2\theta)\right)$$

1. (13 points) Working hard.

(a) Let $\mathbf{F}(x,y,z) = \langle xy^5, zx, 1\rangle$ and C be the curve parameterized by $\mathbf{r}(t) = \langle 3\cos(t), 7t, 3\sin(t)\rangle$ where $-\pi \leq t \leq 4\pi$.

Set up but **do not** evaluate the line integral: $\displaystyle\int_C \mathbf{F} \bullet d\mathbf{r}$.

(b) Let C be the line $y = 2x - 1$ where $1 \leq x \leq 2$. Compute $\displaystyle\int_C (y+1)\,dx + x\,dy$.

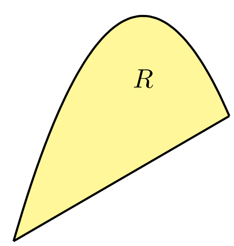
2. (12 points) Let R be the region bounded by $y = 5 - x^2$ and $y = x - 1$ (pictured below).

[*Warning:* One of the integrals below will have to be split into 2 pieces.]

(a) Set up the integral $\displaystyle\iint_R 2xy\,dA$ in the order of integration: "$dy\,dx$".

(b) Set up the integral $\displaystyle\iint_R 2xy\,dA$ in the order of integration: "$dx\,dy$".

Do not evaluate these integrals.



3. (12 points) Compute $\int_0^2 \int_{y/2}^1 \cos(x^2)\, dx\, dy$. Include a **sketch** of **the region** of integration.

[*Hint:* You cannot integrate $\int \cos(x^2)\, dx$ in terms of elementary functions.]

4. (13 points) Set up but **do not** compute $\iint_R \frac{-3x+y}{x+y}\, dA$ where R is the region bounded by

$y = 3x+1$, $y = 3x+2$, $y = -x+1$, and $y = -x+3$.

Hint: Use a "natural" change of coordinates which simplifies the region R and... don't forget the Jacobian!

5. (13 points) Let R be the region bounded by $x^2+y^2=1$ and $x^2+y^2=4$ such that $x \geq 0$. Note: The area of R is $\frac{3}{2}\pi$.

Centroid formulas: $$m = \iint\limits_R 1\,dA \qquad M_y = \iint\limits_R x\,dA \qquad M_x = \iint\limits_R y\,dA \qquad (\bar{x},\bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

(a) Sketch R and fill in the (polar) bounds for R: $\_\_\_\_\_ \leq r \leq \_\_\_\_\_$ and $\_\_\_\_\_ \leq \theta \leq \_\_\_\_\_$.

(b) Find the centroid of R.

6. (12 points) Consider the integral: $I = \displaystyle\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{0} z\cdot(x^2+y^2+z^2)\,dz\,dy\,dx.$

(a) Rewrite I in the following order of integration: $\iiint \quad dx\,dz\,dy$.

Do **not** evaluate the integral.

(b) Rewrite I in terms of cylindrical coordinates.

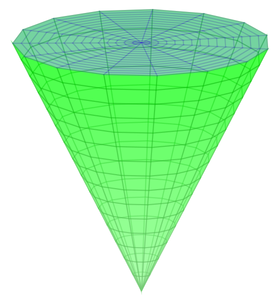
Do **not** evaluate the integral.

(c) Rewrite I in terms of spherical coordinates.

Do **not** evaluate the integral.

7. (10 points) Compute $\iiint_E x^2 + y^2 \, dV$ where E is the region bounded by $z = -1$, $z = 4$, and $x^2 + y^2 = 4$.

8. (15 points) Let E be the region below $z = 6$ and above $z = 2\sqrt{x^2 + y^2}$. Set up integrals which compute the volume of E using the following orders of integration: [Do **not** evaluate these integrals.]



(a) $\int_?^? \int_?^? \int_?^? ??? \, dz \, dy \, dx$

(b) Set up this integral in cylindrical coordinates.

(c) Set up this integral in spherical coordinates.