Name ________________________________ Math 251, 01-03, Exam #1 Oct., 2005

Be sure to show all your work. Unsupported answers will receive no credit.

A formula sheet is supplied, for your reference.

Use the backs of the exam pages for scratchwork or for continuation of your answers, if necessary.

Problem No.	**Pts Possible**	**Points**
1	10	
2	10	
3	12	
4	10	
5	12	
6	12	
7	12	
8	10	
9	12	
Total	100	

1. (10 points): Consider the function

$$f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

(a) Is $f(x,y)$ continuous at the origin?
Hint: Use polar coordinates to evaluate a limit.

(b) Where is $f(x,y)$ continuous?

2. (10 points): Show that $\dfrac{\mathrm{d}}{\mathrm{d}t}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}(t) \times \mathbf{r}''(t)$.

3. (12 points): Let $f(x,y) = xe^{xy}$.
(a) Find $\mathbf{D_u}\, f$ at $(x,y) = (1,0)$ where $\mathbf{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$.

(b) What is the maximal value of $\mathbf{D_u}\, f$ at $(x,y) = (1,0)$?

(c) Find $\mathbf{u}$ such that $\mathbf{D_u}\, f$ takes on this maximal value at $(x,y) = (1,0)$.

4. (10 points): Let $\ln(x + 2y + 3z) = 4$. Find $\dfrac{\partial z}{\partial x}$.

5. (12 points): Consider the paraboloid $z = x^2 + y^2$.
(a) Find the tangent plane at the point $(1, -1, 2)$.

(b) Find parametric equations for the normal line at the point $(1, -1, 2)$.

6. (12 points):
(a) Find parametric equations for the line through the points $(-4, -6, 1)$ and $(-2, 0, -3)$.

(b) Find parametric equations for the line through the points $(10, 18, 4)$ and $(5, 3, 14)$.

(c) Are these lines parallel? Why? or Why not?

7. (12 points): Find the curvature of $\mathbf{r}(t) = \left\langle \frac{t^5}{5}, \frac{2t^3}{3}, t+1 \right\rangle$ when $t = 1$.

8. (10 points): Let $z = f(x, y)$, $x(t) = e^t$, and $y(t) = e^{2t}$. Find $\frac{\mathrm{d}z}{\mathrm{d}t}$.

9. (12 points): Consider the curve $\mathbf{r}(t) = \left\langle 2t, t^2, \frac{t^3}{3} \right\rangle$. Find $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$ when $t = 0$.

Formulas for Exam #1

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin(x) = \cos(x) \qquad \frac{\mathrm{d}}{\mathrm{d}x}a^x = a^x \ln(a)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos(x) = -\sin(x) \qquad \frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$$

$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta \qquad |\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$$

The vector equation for a line with direction $\mathbf{v}$ which passes through a point $\mathbf{r_0}$ is:

$$\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$$

The equation of a the plane tangent to $z = f(x, y)$ at the point (x_0, y_0, z_0) where $z_0 = f(x_0, y_0)$ is:

$$(z - z_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

The distance from the point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is:

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$\mathrm{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} \qquad \mathrm{proj}_{\mathbf{a}}\mathbf{b} = \frac{(\mathbf{a}\cdot\mathbf{b})}{(\mathbf{a}\cdot\mathbf{a})}\mathbf{a}$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c} \qquad \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}$$

$$s(t) = \int_a^t |\mathbf{r}'(t)|\,\mathrm{d}t$$

Curvature for $\mathbf{r}(t)$

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

Curvature for a plane curve $y = f(x)$

$$\kappa = \frac{|f''(x)|}{[1 + (f'(x))^2]^{\frac{3}{2}}}$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} \qquad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} \qquad \mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$$

$$a_{\mathbf{N}} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$$

$$a_{\mathbf{T}} = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|}$$

Consider the level surface $F(x, y, z) = K$, K a constant. Then,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$\nabla f(x, y, z) = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$$