Name ______________________________ Math 251, 01-03, Exam #2 Nov., 2005

Be sure to show all your work. Unsupported answers will receive no credit.

A formula sheet is supplied, for your reference.

Use the backs of the exam pages for scratchwork or for continuation of your answers, if necessary.

Problem No.	**Pts Possible**	**Points**
1	12	
2	10	
3	12	
4	12	
5	11	
6	11	
7	11	
8	10	
9	11	
Total	100	

1. (12 points): Let $\mathbf{F}(x, y, z) = (2x + yz)\mathbf{i} + (z\cos(y) + xz)\mathbf{j} + (\sin(y) + xy - 3z^2)\mathbf{k}$. Show that $\mathbf{F}$ is conservative by finding a function f such that $\mathbf{F} = \nabla f$.

2. (10 points): Let $f(x, y) = x^2 + 2y^2 + xy^2 + 1$. Find all critical points, then determine whether each critical point is a minimum, a maximum, a saddle point, or a nothing.

3. (12 points): Use the method of Lagrange multipliers to find the maximum and minimum of $f(x, y) = xy$ subject to the constraint $x^2 + y^2 = 4$. **WARNING:** You must use Lagrange multipliers!

4. (12 points): Evaluate the following integral using an *obvious* change of variables:

$$\iint_R \frac{2x + y}{x - y}\, dA$$

where R is the parallelogram bounded by the lines $2x + y = 1$, $2x + y = 0$, $x - y = 2$, and $x - y = 1$.

5. (11 points): Evaluate the following integral (*Hint:* reverse the order of integration):

$$\int_0^4 \int_{\sqrt{x}}^2 3\sqrt{1+y^3}\, dy\, dx$$

6. (11 points): Consider the solid E which is bounded by $z = \sqrt{9 - x^2 - y^2}$ and the xy-plane. Write a triple integral which computes the volume of E in: rectangular, cylidrical, and spherical coordinates. Then find the volume of E. (*Hint:* If you identify this solid, you can use simple highschool geometry to find its volume.)

7. (11 points): Evaluate the following integral (*Hint:* change coordinates!):

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \frac{e^{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}\, dz\, dy\, dx$$

8. (10 points): Let $\mathbf{F}(x, y) = e^{-y}\mathbf{i} + (2y - xe^{-y})\mathbf{j}$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is any curve from $(0, 0)$ to $(1, 2)$.

9. (11 points): Find the center of mass of a thin wire with constant density $\rho(x, y) = 2$ in the shape of the upper half of the circle of radius 2 centered at the origin.

Formulas for Exam #2

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin(x) = \cos(x) \qquad \frac{\mathrm{d}}{\mathrm{d}x}a^x = a^x\ln(a)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos(x) = -\sin(x) \qquad \frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$$

$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta \qquad |\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$$

$$\nabla f(x,y,z) = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$$

$$D = f_{xx}f_{yy} - f_{xy}^2 \quad \begin{array}{llll} D>0 & \text{and} & f_{xx}>0 & \text{implies local minimum} \\ D>0 & \text{and} & f_{xx}<0 & \text{implies local maximum} \\ D<0 & & & \text{implies saddle point} \end{array}$$

Center of mass - 3 dimensions

$$\text{mass} = m = \iiint_E \rho(x,y,z)\,dV \qquad (\bar{x},\bar{y},\bar{z}) = \frac{1}{m}(M_{yz}, M_{xz}, M_{xy})$$

$$M_{yz} = \iiint_E x\,\rho(x,y,z)\,dV, \qquad M_{xz} = \iiint_E y\,\rho(x,y,z)\,dV, \qquad M_{xy} = \iiint_E z\,\rho(x,y,z)\,dV$$

Center of mass - a wire in the plane

$$\text{mass} = m = \int_C \rho(x,y)\,ds \qquad (\bar{x},\bar{y}) = \frac{1}{m}(M_y, M_x)$$

$$M_y = \int_C x\,\rho(x,y)\,ds, \qquad M_x = \int_C y\,\rho(x,y)\,ds$$

Polar coordinates:

$$x = r\cos(\theta), \qquad y = r\sin(\theta), \qquad dA = r\,drd\theta$$

Cylindrical coordinates:

$$x = r\cos(\theta), \qquad y = r\sin(\theta), \qquad z = z, \qquad dV = r\,drd\theta dz$$

Spherical coordinates:

$$x = \rho\sin(\phi)\cos(\theta), \qquad y = \rho\sin(\phi)\sin(\theta), \qquad z = \rho\cos(\phi), \qquad dV = \rho^2\sin(\phi)\,d\rho d\theta d\phi$$

Change of variables:

$$\iint_R f(x,y)\,dx\,dy = \iint_S f(x(u,v),y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,du\,dv$$

$$\iiint_R f(x,y,z)\,dx\,dy\,dz = \iiint_S f(x(u,v,w),y(u,v,w),z(u,v,w))\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right|\,du\,dv\,dw$$