Name ________________________________ Math 251, Sec. 5–7, Exam #1 Feb., 2007

Be sure to show all your work. Unsupported answers will receive no credit.

Use the backs of the exam pages for scratchwork or for continuation of your answers, if necessary.

Problem No.	**Pts Possible**	**Points**
1	11	
2	12	
3	10	
4	10	
5	10	
6	10	
7	15	
8	12	
9	10	
Total	100	

1. (11 points): Consider the function

$$f(x,y) = \begin{cases} \dfrac{x^2+y^2}{x^2+4y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

(a) Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? If so, find it. If not, show that it does not exist.

(b) Is $f(x,y)$ continuous at the point $(0,0)$? Why? or Why not?

(c) Is $f(x,y)$ continuous at the point $(1,1)$? Why? or Why not?

2. (12 points): Consider the following lines:

$$\text{Line } L_1: \quad \begin{array}{rcrcr} x(t) & = & 1 & + & t \\ y(t) & = & 2 & - & t \\ z(t) & = & 3 & + & t \end{array} \qquad\qquad \text{Line } L_2: \quad \begin{array}{rcrcr} x(t) & = & & & 3t \\ y(t) & = & -2 & + & 2t \\ z(t) & = & 4 & + & t \end{array}$$

(a) Are L_1 and L_2 parallel, intersecting, or skew lines?

(b) Find a plane which contains the line L_1 and is parallel to the line L_2.

3. (10 points): Consider the curve $\mathbf{r}(t) = \langle e^t \cos(t), \sqrt{2}e^t, e^t \sin(t) \rangle$ where $-\pi \leq t \leq \pi$. Find parametric equations for the tangent line of $\mathbf{r}(t)$ at $t = 0$.

4. (10 points): Let $z = f(x, y)$ where $x = s + t$ and $y = s - t$. Assuming that f is differentiable, show that:

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \frac{\partial z}{\partial s} \cdot \frac{\partial z}{\partial t}.$$

Hint: Compute $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ using the chain rule first.

5. (10 points): Random dot and cross product questions.

(a) If $\mathbf{a}$ is a unit vector in $\mathbb{R}^2$ and $\mathbf{a} \cdot \langle 1, 0 \rangle = 0$, what could $\mathbf{a}$ be? Is there more than one answer?

(b) Let $\mathbf{a}$ and $\mathbf{b}$ be vectors in $\mathbb{R}^3$ such that $\mathbf{a} \times \mathbf{b} = \mathbf{0}$. What can we say about $\mathbf{a}$ and $\mathbf{b}$?

6. (10 points): Find the plane tangent to the level surface $x^2 - y^2 + z^2 = 1$ at the point $(-1, 0, 0)$.

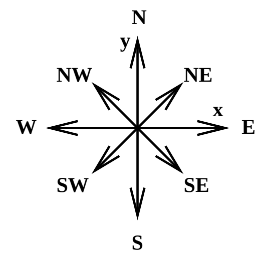
7. (15 points): Consider the curve: $\mathbf{r}(t) = \langle 5\cos(t), 5\sin(t), 0\rangle$ where $0 \leq t \leq 2\pi$.

(a) Find a formula for the arc length function $s(t)$ of $\mathbf{r}(t)$ and then use it to reparametrize $\mathbf{r}(t)$ with respect to arc length.

(b) Find $\mathbf{T}(t)$, $\mathbf{N}(t)$, and $\mathbf{B}(t)$ for this curve.

(c) Find the curvature $\kappa(t)$ of this curve.

8. (12 points): You are standing in a desert whose temperature (in degrees fahrenheit) is described by the function $f(x, y) = 105 + 15\sin(x + y)$. Your current location is $(0, 0)$. Note: $(1, 0)$ is 1 mile East of your location, $(-1, 0)$ is 1 mile West, $(0, 1)$ is 1 mile North, and $(0, -1)$ is 1 mile South.



(a) Use a directional derivative to measure the change in temperature if you start walking North.

(b) Walking in what direction will give a maximal *decrease* in temperature?

9. (10 points): Find all of the second partials of $z = x^2 \ln(y)$.

Formulas for Exam #1

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin(x) = \cos(x) \qquad \frac{\mathrm{d}}{\mathrm{d}x}a^x = a^x\ln(a)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos(x) = -\sin(x) \qquad \frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$$

$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta \qquad |\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$$

The vector equation for a line with direction $\mathbf{v}$ which passes through a point $\mathbf{r_0}$ is:

$$\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$$

The equation of a the plane tangent to $z = f(x, y)$ at the point (x_0, y_0, z_0) where $z_0 = f(x_0, y_0)$ is:

$$(z - z_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

The distance from the point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is:

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$\mathrm{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} \qquad \mathrm{proj}_{\mathbf{a}}\mathbf{b} = \frac{(\mathbf{a}\cdot\mathbf{b})}{(\mathbf{a}\cdot\mathbf{a})}\mathbf{a}$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c} \qquad \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}$$

$$s(t) = \int_a^t |\mathbf{r}'(t)|\,\mathrm{d}t$$

Curvature for $\mathbf{r}(t)$

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

Curvature for a plane curve $y = f(x)$

$$\kappa = \frac{|f''(x)|}{[1 + (f'(x))^2]^{\frac{3}{2}}}$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} \qquad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} \qquad \mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$$

$$a_{\mathbf{N}} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$$

$$a_{\mathbf{T}} = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|}$$

Consider the level surface $F(x, y, z) = K$, K a constant. Then,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$\nabla f(x, y, z) = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$$