Name ________________________________ Math 251, Sec. 5–7, Exam #2 Apr., 2007

Be sure to show all your work. Unsupported answers will receive no credit.

Use the backs of the exam pages for scratchwork or for continuation of your answers, if necessary.

Problem No.	**Pts Possible**	**Points**
1	12	
2	12	
3	14	
4	13	
5	12	
6	11	
7	13	
8	13	
Total	100	

1. (12 points): Consider the function $f(x, y) = x^3 + x^2y - y^2 - 4y$. Find all of the critical points of $f(x, y)$ and classify them (i.e. as a minimum, maximum, saddle point, or nothing).

2. (12 points): Let C be the line segment from $(1, 2, 3)$ to $(-1, 0, 1)$.

Evaluate $\int_C z^2 - x^2 \, ds$.

3. (14 points): Fix two positive real numbers a and b.
Let $x = ar\cos(\theta)$ and $y = br\sin(\theta)$.

(a) Compute $\dfrac{\partial(x, y)}{\partial(r, \theta)}$.

(b) Let $R = \left\{ (x, y) \mid \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} \leq 1 \right\}$. Find the area of R by evaluating a double integral. *Hint:* Use the modified polar coordinates defined above.

4. (13 points): Let E be the region inside both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$. Evaluate $\displaystyle\iiint_E 2(x^2 + y^2)z \, dV$.

5. (12 points): Use Lagrange multipliers to find the maximum and minimum value of $f(x, y, z) = xyz$ subject to the constraint $x^2 + y^2 + z^2 = 3$.

6. (11 points): Evaluate $\int_0^1 \int_x^1 e^{y^2}\, dy\, dx$.

7. (13 points): Consider the vector field $\mathbf{F}(x, y) = \left(2xe^{2y} + 3x^2\right)\mathbf{i} + \left(2x^2e^{2y} + \cos(y)\right)\mathbf{j}$.

(a) Show that $\mathbf{F}(x, y)$ is **conservative** by finding a potential function.

(b) Let C be the upper-half of the circle $x^2 + y^2 = 1$ oriented counter-clockwise. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

8. (13 points): Find the centroid of the region, E, inside the sphere $x^2 + y^2 + z^2 = 4$ and above the xy-plane. *Hint:* The volume of half of a sphere or radius r is $\frac{2}{3}\pi r^3$. Use this to find m. Also, $\bar{x} = \bar{y} = 0$ by symmetry.

Formulas for Exam #2

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin(x) = \cos(x) \qquad \frac{\mathrm{d}}{\mathrm{d}x}a^x = a^x\ln(a)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos(x) = -\sin(x) \qquad \frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$$

$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta \qquad |\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$$

$$\nabla f(x,y,z) = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$$

$$D = f_{xx}f_{yy} - f_{xy}^2 \quad \begin{array}{lll} D>0 & \text{and} \quad f_{xx}>0 & \text{implies local minimum} \\ D>0 & \text{and} \quad f_{xx}<0 & \text{implies local maximum} \\ D<0 & & \text{implies saddle point} \end{array}$$

Center of mass - 3 dimensions

$$\text{mass} = m = \iiint_E \rho(x,y,z)\,dV \qquad (\bar{x},\bar{y},\bar{z}) = \frac{1}{m}(M_{yz}, M_{xz}, M_{xy})$$

$$M_{yz} = \iiint_E x\,\rho(x,y,z)\,dV, \qquad M_{xz} = \iiint_E y\,\rho(x,y,z)\,dV, \qquad M_{xy} = \iiint_E z\,\rho(x,y,z)\,dV$$

Center of mass - a wire in the plane

$$\text{mass} = m = \int_C \rho(x,y)\,ds \qquad (\bar{x},\bar{y}) = \frac{1}{m}(M_y, M_x)$$

$$M_y = \int_C x\,\rho(x,y)\,ds, \qquad M_x = \int_C y\,\rho(x,y)\,ds$$

Polar coordinates:

$$x = r\cos(\theta), \qquad y = r\sin(\theta), \qquad dA = r\,drd\theta$$

Cylindrical coordinates:

$$x = r\cos(\theta), \qquad y = r\sin(\theta), \qquad z = z, \qquad dV = r\,drd\theta dz$$

Spherical coordinates:

$$x = \rho\sin(\phi)\cos(\theta), \qquad y = \rho\sin(\phi)\sin(\theta), \qquad z = \rho\cos(\phi), \qquad dV = \rho^2\sin(\phi)\,d\rho d\theta d\phi$$

Change of variables:

$$\iint_R f(x,y)\,dx\,dy = \iint_S f(x(u,v),y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,du\,dv$$

$$\iiint_R f(x,y,z)\,dx\,dy\,dz = \iiint_S f(x(u,v,w),y(u,v,w),z(u,v,w))\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right|\,du\,dv\,dw$$

Line Integrals : $\quad ds = |\mathbf{r}'(t)|dt \qquad dx = x'(t)dt \qquad d\mathbf{r} = \mathbf{r}'(t)dt$