Name ________________________________ Math 251, Sec. H1, Exam #2 Nov., 2007

Be sure to show all your work. Unsupported answers will receive no credit.

Use the backs of the exam pages for scratchwork or for continuation of your answers, if necessary.

Problem No.	**Pts Possible**	**Points**
1	10	
2	10	
3	12	
4	13	
5	12	
6	14	
7	14	
8	15	
Total	100	

1. (10 points): Find all of the critical points of $f(x, y) = 4xy - x^4 - y^4$. Determine whether each point is a relative minimum, relative maximum, or saddle point.

2. (10 points): Use the method of "Lagrange multipliers" to find three *non-negative* numbers whose sum is 18 and whose product is as large as possible.

3. (12 points): Consider the integral: $\int_0^1 \int_{4x}^4 e^{-y^2}\, dy\, dx$.

(a) Sketch the region of integration.

(b) Evaluate the integral.

4. (13 points): Consider the triple integral $\iiint_E f(x, y, z)\, dV$ where E is the solid inside the sphere $x^2 + y^2 + z^2 = 4$ and to the right of $y = 0$ (that is $y \geq 0$).

(a) Express the above triple integral as an interated integral in the following orders of integration:

$$\int \quad \int \quad \int \quad f(x, y, z) dz\, dy\, dx$$

$$\int \quad \int \quad \int \quad f(x, y, z) dx\, dy\, dz$$

$$\int \quad \int \quad \int \quad f(x, y, z) dy\, dz\, dx$$

(b) Rewrite the integral in cylindrical coordinates – *this answer should not have any x's or y's!*

(c) Rewrite the integral in spherical coordinates – *this answer should not have any x's, y's, or z's!*

5. (12 points): Evaluate $\displaystyle\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} \frac{1}{\sqrt{x^2+y^2+z^2}}\,dz\,dy\,dx$

6. (14 points): Let D be the region bounded by the lines: $y = x/2$, $x + y = 1$, and $y = 0$. Evaluate the double integral $\iint_D \sqrt{\frac{x+y}{x-2y}}\, dA$ by changing variables. Please include sketches of both the region D and the new region obtained after changing variables. *Don't forget the Jacobian!*

7. (14 points): A wire is bent into a semicircular shape described by $x^2 + y^2 = 4$, $-2 \leq x \leq 2$, and $y \geq 0$. Suppose that the wire has constant density (set $\rho = 1$).

(a) Parameterize and sketch the curve $x^2 + y^2 = 4$, $-2 \leq x \leq 2$, $y \geq 0$. Then determine it's length two ways: (i) Using highschool geometry and (ii) Using a line integral.

(b) Find the wire's center of mass.

8. (15 points): Let $\mathbf{F}(x, y, z) = \langle 2x + yz\cos(x), z\sin(x) + z^2, y\sin(x) + 2yz + 3z^2 \rangle$.

(a) Is $\mathbf{F}(x, y, z)$ conservative? If so, find a potential function for $\mathbf{F}(x, y, z)$. If not, explain why $\mathbf{F}(x, y, z)$ is not conservative.

(b) Let C be the curve $\mathbf{r}(t) = \langle t, \sin(t), \cos(t) - 1 \rangle$ where $0 \leq t \leq 2\pi$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}$ is the vector field from part (a).

(c) Let $\mathbf{G}(x, y) = \langle P(x, y), Q(x, y) \rangle$ be a conservative vector field and let C be some **closed curve** such that $\int_C P(x, y)\, dx = 5$. Find $\int_C Q(x, y)\, dy$ and explain your answer.

Formulas for Exam #2

$$\nabla f(x,y,z) = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$$

$$D = f_{xx}f_{yy} - f_{xy}^2 \quad \begin{array}{llll} D>0 & \text{and} & f_{xx}>0 & \text{implies local minimum} \\ D>0 & \text{and} & f_{xx}<0 & \text{implies local maximum} \\ D<0 & & & \text{implies saddle point} \end{array}$$

Center of mass - 2 dimensions

$$\text{mass} = m = \iint_R \rho(x,y)\,dV \qquad (\bar{x},\bar{y}) = \frac{1}{m}(M_y, M_x)$$

$$M_y = \iint_R x\,\rho(x,y)\,dA \quad \text{and} \quad M_x = \iint_R y\,\rho(x,y)\,dA$$

Center of mass - 3 dimensions

$$\text{mass} = m = \iiint_E \rho(x,y,z)\,dV \qquad (\bar{x},\bar{y},\bar{z}) = \frac{1}{m}(M_{yz}, M_{xz}, M_{xy})$$

$$M_{yz} = \iiint_E x\,\rho(x,y,z)\,dV, \quad M_{xz} = \iiint_E y\,\rho(x,y,z)\,dV, \quad M_{xy} = \iiint_E z\,\rho(x,y,z)\,dV$$

Center of mass - a wire in the plane

$$\text{mass} = m = \int_C \rho(x,y)\,ds \qquad (\bar{x},\bar{y}) = \frac{1}{m}(M_y, M_x)$$

$$M_y = \int_C x\,\rho(x,y)\,ds \quad \text{and} \quad M_x = \int_C y\,\rho(x,y)\,ds$$

Polar coordinates:

$$x = r\cos(\theta), \qquad y = r\sin(\theta), \qquad dA = r\,dr d\theta$$

Cylindrical coordinates:

$$x = r\cos(\theta), \qquad y = r\sin(\theta), \qquad z = z, \qquad dV = r\,drd\theta dz$$

Spherical coordinates:

$$x = \rho\sin(\varphi)\cos(\theta), \qquad y = \rho\sin(\varphi)\sin(\theta), \qquad z = \rho\cos(\varphi), \qquad dV = \rho^2\sin(\varphi)\,d\rho d\theta d\varphi$$

Change of variables:

$$\iint_R f(x,y)\,dx\,dy = \iint_S f(x(u,v), y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,du\,dv$$

$$\iiint_R f(x,y,z)\,dx\,dy\,dz = \iiint_S f(x(u,v,w), y(u,v,w), z(u,v,w))\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right|\,du\,dv\,dw$$

$$\text{Line Integrals}: \qquad ds = |\mathbf{r}'(t)|dt \qquad dx = x'(t)dt \qquad d\mathbf{r} = \mathbf{r}'(t)dt$$