Math 2510 # Test #1 **February 23^{rd}, 2010**

Name: ________________________________

Don't merely state answers, prove your statements. **Be sure to show your work!**

1. (____/12 points) Consider theorem L10: $\vdash (\neg B \to \neg A) \to (A \to B)$

(a) Show L10 is a tautology using by filling out an abbreviated truth table.

(b) Prove L10 in system L. [You may use the deduction theorem and theorems L1 – L9.]

2. (____/12 points) Flawed proof

I am in the middle of proving "$A \to B, \neg A \to B \vdash B$". First, show (using an abbreviated truth table) that this is indeed a theorem of L. Then explain why I know I've made a mistake if I have the following lines in my proof:

$\vdots$	$\vdots$	$\vdots$
10	A	M.P. (lines 5 and 9)
11	$\neg\neg B \to \neg A$	M.P. (lines 2 and 7)
12	$(\neg\neg B \to \neg A) \to (A \to \neg B)$	L10
$\vdots$	$\vdots$	$\vdots$

3. (____/10 points) Construct models whose objects are $\mathbb{Z}$ (integers) to show that

$$\forall x \forall y\, (P(x,y) \rightarrow P(f(y), f(x)))$$

is satisfiable but not logically valid. [Note: $P(x,y)$ is a predicate and $f(x)$ is a function.]

4. (____/14 points) Proofs in K.

(a) Prove theorem K13: $\vdash \forall x A(x) \to \exists x A(x)$

(b) Prove theorem K31: $\vdash \exists x(A(x) \land B(x)) \to (\exists x A(x) \land \exists x B(x))$

5. (____/14 points) How about...more proofs?

(a) Use induction to show that $1 + 4 + 7 + \cdots + (3n - 2) = \frac{1}{2}n(3n - 1)$ for all positive integers n.

(b) Prove that $\sqrt{2}$ is irrational. [Recall: x is rational means $\exists$ integers p, $q \neq 0$ such that $x = p/q$.]
Hint: Proof by contradiction.

Math 2510 # Test #1 – TAKE HOME **Due: Feb. 24^{th}, 2010**

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You may use notes and your textbook, but no help from other people especially your classmates.

6. (____/12 points) A few more proofs in L

(a) Prove theorem L7: $A \to (B \to C) \vdash B \to (A \to C)$. You may use the deduction theorem and theorems L1 – L6.

(b) Prove $\vdash (A \wedge \neg A) \to B$. You may use the deduction theorem and theorems L1 – L15.

(c) Prove $A \vee B \vdash B \vee A$. You may use the deduction theorem and theorems L1 – L12.

7. (____/15 points) A few more proofs in K. Prove (at least) 5 of the theorems K27 – K37.

8. (____/12 points) More proofs !?!

(a) Prove that $1 \cdot 1! + 2 \cdot 2! + \cdots n \cdot n! = (n+1)! - 1$ for all positive integers n.

(b) Let n be an odd integer. Prove that $n^2 - 1$ is a multiple of 4.

(c) Let r be an irrational number and m be an integer. Prove that mr is irrational.

(d) Let a, b, and c be integers. Show that if $a \,\Big|\, b$ and $b \,\Big|\, c$, then $a \,\Big|\, c$.