Math 2510 # Test #2 **April 2^{nd}, 2010**

Name: ________________________________

Don't merely state answers, prove your statements. **Be sure to show your work!**

1. (____/28 points) Converging Questions

(a) Prove that $\left\langle \dfrac{3n^2}{n^2+2n+3} \right\rangle$ converges.

(b) Show that $\left\langle \dfrac{\sin(n)}{n^4+1} \right\rangle$ converges.
Hint: Ignore $\sin(n)$, then prove $\sin(n)$ is bounded and use a theorem.

(c) Prove that $\left\langle \frac{n^2+1}{n} \right\rangle$ diverges.

(d) Let $a_n \to A$ and $b_n \to B$. Show that $a_n + b_n \to A + B$.

2. (____/20 points) Some set stuff.

(a) Let A, B, C, D be sets. Suppose that $A \cup B \subseteq C \cup D$, $A \cap B = \phi$, and $C \subseteq A$. Prove that $B \subseteq D$.

(b) Let $f : X \to Y$ and let $T \subseteq Y$. Prove that if f is onto, then $f(f^{-1}(T)) = T$. Point out which "half" of your proof does **not** need the "onto" hypothesis.

3. (____/28 points) For each of the following functions, decide if f is 1-1, onto, both, or neither. **Prove your answers!**

(a) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 5x + 2$

(b) Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = 5x + 2$

(c) Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = \begin{cases} 2x & x \text{ is even} \\ x+1 & x \text{ is odd} \end{cases}$

(d) Let $g : X \to Y$ and $f : Y \to Z$. Assume that $f \circ g : X \to Z$ is a bijection. If f **must be** 1-1, prove it. If f might not be 1-1, give a counter-example. If f **must be** onto, prove it. If f might not be onto, give a counter-example. [A counter-example should specify **both** f and g and be accompanied by a proof that it is in fact a counter-example.]

4. (____/28 points) Equivalent Nonsense.

Recall that for integers a and b, $a \cong b \pmod 4$ if and only if there exists some $k \in \mathbb{Z}$ such that $a = b + 4k$.

(a) Prove that $a \cong b \pmod 4$ is an equivalence relation on $\mathbb{Z}$.

(b) List the equivalence classes of this equivalence relation.

(c) Prove that the function $f : \mathbb{Z}_4 \to \mathbb{Z}_{10}$ defined by $f([n]) = [5n]$ is well-defined. [$\mathbb{Z}_m$ are the equivalence classes of integers mod m.]

(d) Let $n \in \mathbb{Z}$ be a sum of squares. [That is there exist integers a and b such that $n = a^2 + b^2$.] Prove that $n \not\equiv 3 \pmod 4$.