Math 2510 **Test #1** **February 27^{th}, 2015**

Name: ______________________________

Don't merely state answers, prove your statements. **Be sure to show your work!**

1. (10 points) Consider theorem L15: $\vdash \neg A \to (A \to B)$

(a) Show L15 is a tautology using by filling out an abbreviated truth table.

(b) Prove L15 in system L. [You may use the deduction theorem and theorems L1 – L14.]
Hint: I have a quick proof using the deduction theorem and lemmas L5 and L10 in mind.

2. (10 points) Still in System L...

(a) Is "$A \vee B \vdash A$" provable in L? What about "$A \vdash A \vee B$"? Justify your answer(s).

(b) Here is my proof of theorem L7: $A \to (B \to C) \vdash B \to (A \to C)$...

1: $A \to (B \to C)$ 1: ______

2: $B \to (A \to (B \to C))$ 2: ______

3: $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$ 3: ______

4: $(A \to B) \to (A \to C)$ 4: ______

5: $B \to ((A \to B) \to (A \to C))$ 5: ______

6: $B \to (A \to B)$ 6: ______

7: $B \to (A \to C)$ 7: L3 with $A :=$ ______, $B :=$ ______, and $C :=$ ______

Fill in justifications for each line. I used the usual rules of system L and restricted myself to using lemmas among L1 – L6.

3. (10 points) Construct models whose objects are $\mathbb{Z}$ (integers) to show that

$$(\forall x\, P(x)) \rightarrow (\exists y\, Q(\underline{c}, f(y)))$$

is satisfiable but not logically valid. [Note: $P(x)$ and $Q(x, y)$ are predicates, $f(x)$ is a function, and $\underline{c}$ is a constant.]

4. (10 points) Proofs in K. You may use the deduction theorem and lower numbered theorems.

(a) Prove theorem K7: $\vdash \forall x(A(x) \to B(x)) \to (\forall x A(x) \to \forall x B(x))$

(b) Prove theorem K21: $\vdash \exists x A(x) \to \exists x(A(x) \lor B(x))$

5. (10 points) How about...more proofs?

(a) Use induction to show that $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$ for all positive integers n.

(b) Use proof by contradiction to show: For all $x, y \in \mathbb{Z}$, if xy is even, then either x or y is even.

Math 2510 # Test #1 – TAKE HOME **Due: Mar. 2^{nd}, 2015**

Name: ____________________________________

You may use notes and your textbook, but no help from other people [except myself and Noah] especially your classmates.

6. (25 points) Redo the in-class portion of the test.

7. (15 points) A few more proofs in K. Prove (at least) 5 of the theorems K27 – K37.

8. (15 points) More proofs !?!

(a) Prove that $n! > 2^n$ for all integers $n \geq 4$.

(b) Prove that $\sqrt[3]{2}$ is irrational.

(c) For any integer n, prove that $n^2 + 2$ is not divisible by 4.