Math 3110 **Test #2** **October 28^{th}, 2009**

Name: ___________________________________

Don't merely state answers, prove your statements.

1. (____/16 points) Either prove G is a group or explain why it is not a group.

(a) $G = \{x \in D_5 \,|\, x \text{ is a reflection}\}$ (the operation is composition of symmetries).

(b) $\mathbb{R}_{>0}$ (positive real numbers) where the operation is multiplication.

2. (_____/16 points) (a) Show $H = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \,\middle|\, a, b \in \mathbb{R} \text{ and } a, b \neq 0 \right\}$ is a subgroup of $\mathrm{GL}_2(\mathbb{R})$

(Recall $\mathrm{GL}_2(\mathbb{R})$ — the set of 2×2 invertible matrices — is a group under matrix multiplication).

(b) Quickly (in a few words) why are the even integers a subgroup of the integers?

3. (____/20 points) Workin' mod 5.

(a) Fill out the following tables (don't worry about brackets for equivalence classes.)

$(\mathbb{Z}_5, +)$	0	1	2	3	4
0					
1					
2					
3					
4					

$\mathbb{Z}_5$ Addition Table

$(\mathbb{Z}_5, \times)$	0	1	2	3	4
0					
1					
2					
3					
4					

$\mathbb{Z}_5$ Multiplication Table

(b) Compute $2^{-1}(4+3) - 2 \pmod 5$.

(c) Find $\langle 3 \rangle$ (the subgroup generated by 3) in $U(5)$ (NOT $\mathbb{Z}_5$!!!).

(d) Find the orders of elements of $U(5)$. Is $U(5)$ cyclic? Why or why not?

element =	
order =	

4. (____/16 points) Quick proofs

(a) Let G be a group and suppose that $x = x^{-1}$ for all $x \in G$. Prove that G is Abelian.
Hint: $xy = (xy)^{-1} =$??

(b) Let n be an integer such that $n \geq 3$. Consider $(12), (13) \in S_n$. Compute $(12)(13)$ and $(13)(12)$.
Is S_n cyclic? Why or why not?

5. (____/16 points) $G = \{1, a, a^2, b, ab, a^2b\}$ is a group.

Finish filling out G's Cayley table then answer some questions.

G	1	a	a^2	b	ab	a^2b
1	1	a	a^2	b	ab	a^2b
a	a			ab	a^2b	b
a^2	a^2	1			b	ab
b	b	a^2b	ab	1	a^2	
ab	ab	b	a^2b			
a^2b	a^2b	ab		a^2		1

(a) What is the order of a^2? Determine $\langle a^2 \rangle$ (the subgroup generated by a^2).

(b) Is G Abelian? Is G cyclic? Why or why not?

6. (____/16 points) Permutations!

(a) Write $\sigma = (125)(35)(24)(264)$ as a product of disjoint cycles.

(b) What is the order of σ?

(c) Write σ as a product of transpositions. Is σ even or odd?

(d) Let $\tau = (1452)(367)(89)$. What is the order of τ?

(e) Write τ^{26} as the product of disjoint cycles.

(f) What is the order of τ^6?
[Hint: You shouldn't need to compute any powers of τ to answer this question.]